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THE

COMMON SCHOOL ARITHMETIC,

7260

PREPARED FOR THE USE OF ACADEMIES AND
COMMON SCHOOLS

IN

THE UNITED STATES,

AND

ALSO FOR THE USE OF THE YOUNG GENTLEMEN WHO
MAY BE PREPARING TO ENTER THE MILITARY
ACADEMY AT WEST POINT.

BY CHARLES DAVIES,

PROFESSOR OF MATHEMATICS IN THE MILITARY ACADEMY
AT WEST POINT,

AND

AUTHOR OF ELEMENTS OF DESCRIPTIVE GEOMETRY, SURVEYING,
AND A TREATISE ON SHADOWS AND PERSPECTIVE.

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PREFACE.

A correct and accurate knowledge of Arithmetic is one of the most important elements of a liberal or practical education. The public should, therefore, receive with indulgence every attempt that may be made to improve this department of instruction.

The Elementary Treatise, which is here presented, has been prepared with care, though the author by no means indulges the belief that it is free from faults. He has endeavored to render the subject as plain as it is capable of being made, and, at the same time, to treat it as briefly as possible.

The reasons for most of the rules are given. It was not, however, thought best to demonstrate the rules for the extraction of roots, nor that for finding the sum of a geometrical series.

The name, Compound Numbers, which has heretofore been given to all numbers in which the *kind* of unit is expressed, has been changed to that of Denominate Numbers. This change has not been made with any ambitious spirit of innovation, but because it is deemed an improvement. It is not easy to form an idea of what is meant by the term, Compound Number, and especially so, when we find it applied to such numbers as £3, $3, 3*s*. Why is £3 a compound number any more than 3? If it be answered, that £3 is composed or compounded of three single pounds, that does not remove the difficulty, for 3 is also composed of three units 1. Is it not then the better way to call the first a denominate number, and the other a simple number, as is done in § 71.

Mr. Hasler, in his Arithmetic, has called this class of numbers, Denominate Fractions.

Some of the examples in the Rule of Three, and most of those at the end of the book, have been selected from *Hutton's* and *Walkingame's* Arithmetics.

Military Academy,
West Point, July, 1833.

SUGGESTIONS TO TEACHERS.

The first object has been to fix in the mind of the pupil a clear idea of a unit, this being regarded as the basis of all correct knowledge of Arithmetic.

Each subject is divided into sections, and at the end of the subject, a series of questions is appended. All the questions which refer to the same section, are arranged together, and are intended to suggest to the pupil all that he is required to learn from the particular section to which they refer.

The pupil should be able to answer all the questions in his own language, and to understand perfectly the reasons of the answers.

The parts to be committed to memory are printed in Italic, to distinguish them from those parts that are merely explanatory.

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ARITHMETIC.

INTRODUCTION.

I. Every thing which admits of *increase* or *diminution* is called quantity.

II. Quantities are made up of parts. Thus, six apples is a quantity which is made up of six single apples; ten trees is a quantity made up of ten individual trees; and fifteen peaches is a quantity made up of fifteen single peaches.

III. In the classes of natural objects, such as horses, sheep, apples, trees, peaches, &c. one of the species, or kind, is called the *unit of the quantity*. Thus, one apple is the *unit* of the quantity six apples; one tree is the *unit* of the quantity ten trees, and one peach the *unit* of the quantity fifteen peaches.

IV. There are other quantities, besides the classes of natural objects, which are also made up of parts. For example, ten feet in length is a quantity which is made up of ten separate feet; six pounds of tea is a quantity made up of six separate parts of one pound each, and four hours of time is a quantity made up of four separate hours.

V. We can conceive of a quantity only by comparing it with another quantity of the same kind, whose value is fixed and known. Thus, when we speak of the distance ten feet, we are only able to conceive of it, by first knowing the length of one foot, *and then understanding* that this length may be laid *off ten times in the given* distance ten feet. When *we speak of* six pounds of tea, we unders it is such a quantity as comes from mixing six separate pounds together. When we speak of four hours of time, we mean a duration four times longer than one hour.

VI. In each of the above cases we compare the quantity spoken of with a quantity of the same kind, whose value is known. This known value is called the *unit of the quantity*, and is one of the equal parts of which the quantity is made up.

VII. In the examples above, one foot is the unit of the distance ten feet; one pound of tea is the unit of the quantity six pounds of tea, and one hour the unit of the quantity four hours of time.

VIII. Numbers *are the expressions for several things of the same kind: and they show how many times a quantity contains its unit.* Thus, when we say three trees, the number three expresses three things of the same kind: or if we say six pounds of tea, we understand that the quantity six pounds of tea contains its unit one pound, six times.

IX. In numbers, one is the most simple unit.

QUESTIONS.

I. What is quantity?

II. How are quantities made up? Of what is the quantity six apples made up?—ten trees?—fifteen peaches?

III. In the classes of natural objects what is one called? What is the unit of six apples?—Of fifteen peaches?

IV. Of what is the quantity ten feet made up?—Six pounds of tea?—Four hours of time?

V. How do we conceive of a quantity?—How do we conceive of ten feet?—Of six pounds of tea?—Of four hours of time?

VI. What is the known value called? What is it?

VII. What is the unit of the distance ten feet?—Of six pounds of tea?—Of four hours of time?

VIII. What are numbers? What do they show?

What is the most simple unit?

NUMERATION AND NOTATION.

ARITHMETIC.

§ 1. ARITHMETIC treats of numbers. It is both a science and an art.

It is a science, when it explains the nature and properties of numbers; and an art, when it treats of the best methods of using them.

§ 2. In Arithmetic, numbers are expressed by certain characters, called figures.* There are ten of these characters. They are

0 which is called a cipher, or Naught,
1 - - - - One,
2 - - - - Two,
3 - - - - Three,
4 - - - - Four,
5 - - - - Five,
6 - - - - Six,
7 - - - - Seven,
8 - - - - Eight,
9 - - - - Nine.

---

* NOTE.—There is another method called the Roman, in which, numbers are represented by letters; I represents one, V five, X ten, L fifty, C one hundred, D five hundred, &c.

TABLE.

One	I.	Sixty	LX.
Two	II.	Seventy	LXX.
Three	III.	Eighty	LXXX.
Four	IIII. or IV.	Ninety	LXXXX, or XC.
Five	V.	One hundred	C.
Six	VI.	Two hundred	CC.
Seven	VII.	Three hundred	CCC.
Eight	VIII.	Four hundred	CCCC.
Nine	VIIII. or IX.	Five hundred	D.
Ten	X.	Six hundred	DC.
Twenty	XX.	Seven hundred	DCC.
Thirty	*XXX.*	Eight hundred	DCCC.
Forty	*XXXX. or XL.*	Nine hundred	DCCCC.
Fifty	*L.*	One thousand	M.

§ 3. The character 0 is used to denote the absence of a thing. As, if we wished to express by figures that there were no apples in a basket, we should write, the number of apples in the basket is 0. The nine other figures are called, *significant figures.*

1	expresses	a single thing, or a *unit* of a quantity.	
2	,,	two things of the same kind,	or two units.
3	,,	three things	or three units.
4	,,	four things	or four units.
5	,,	five things	or five units.
6	,,	six things	or six units.
7	,,	seven things	or seven units.
8	,,	eight things	or eight units.
9	,,	nine things	or nine units.

§ 4. If we wish to express the number *ten*, we have no separate character for it. We must *combine* the characters already known. This we do by writing 0 on the right hand of the 1. Thus,

10 which is read - - - - - Ten.

This 10 is equal to *ten* of the units expressed by 1. It is, however, but a *single ten*, and in this sense may be regarded as a *unit*, the value of which is *ten times greater* than the unit expressed by 1.

§ 5 When two figures are thus written by the side of each other, the one on the right is called the *place of units*, the other, the *place of tens*, or *units of the second order ;* and *each unit of the second order is equal to ten units of the first order.*

§ 6. When units simply are mentioned, *units of the first order* are always meant :

Two tens, or twenty, are written	20
Three tens, or thirty,	30
Four tens, or forty,	40
Five tens, or fifty,	50
Six tens, or sixty,	60
Seven tens, or seventy,	70
Eight tens, or eighty,	80
Nine tens, or ninety,	90

§ 7. The intermediate numbers between 10 and 20, between 20 and 30, &c. may readily be expressed by considering the tens and units of which they are composed. For example, the number twelve is composed of one ten and two units. It must therefore be written by writing 1 in the place of tens, and 2 in the place of units; thus, - - 12

Eighteen has 1 ten and 8 units, and is written .	18
Twenty five has 2 tens and five units, and is written	25
Thirty seven has 3 tens 7 units, and is written .	37
Fifty four has 5 tens and 4 units, and is written -	54
Eighty nine has 8 tens and 9 units, and is written .	89
Ninety nine has 9 tens and 9 units, and is written -	99

§ 8. In order to express one hundred, or *ten units of the second order*, we have to form a new combination.

It is done thus, - - - - 100

by writing two ciphers on the right of 1. This number is read, one hundred. Now this one hundred expresses 10 *units of the second order*, or *one hundred units of the first order*. But the one hundred is but *an individual hundred*, and in this light may be regarded as a unit of the *third order*.

§ 9. With these combinations all the numbers between 1 and one thousand may be expressed.

For example, in the number three hundred and seventy five, there are 3 hundreds, 7 tens, and 5 units. We are, therefore, to express 3 units of the 3d order, 7 units of the second order, and 5 of the 1st.

huns.	tens	units
3	7	5

Hence we write - - - - 3 7 5

In the number eight hundred and ninety nine, there are 8 units of the 3d order, 9 of the 2d, and 9 of the 1st.

huns.	tens	units
8	9	9

It is written - - - -

In the number four hundred and six, there are 4 units of the 3d order, 0 of the 2d, and 6 of the 1st.

hund.	tens	units
4	0	6

It is written - - - - - 4 0 6

§ 10. To express *ten* units of the 3d order, or one thousand, we form a new combination by writing three ciphers on the right of 1; thus - - 1000

Now, although this thousand expresses one thousand units of the 1st order, it is, nevertheless, but *one single thousand*, and may be regarded as a unit of the 4th order.

Proceeding in this way, we may place as many figures in a row as we please. When so placed, we conclude:

1st. *That the same figure has different values according to the place which it occupies.*

2d. *That counting from the right hand towards the left, the first place is the place of units; the second, the place of tens; the third, the place of hundreds; the fourth, the place of thousands, &c.*

3d. *That ten units of the first place are equal to one unit of the second place; that ten units of the second place are equal to one unit of the third place; that ten units of the third place are equal to one unit of the fourth place; and so on, for places farther to the left.*

§ 11. Expressing numbers by figures is called NOTATION. Reading the order of their places, correctly, when written, is called NUMERATION.

NUMERATION TABLE.*

	6th Period or Period of Quadrillions.	5th Period or Period of Trillions.	4th Period or Period of Billions.	3d Period or Period of Millions.	2d Period or Period of Thousands.	1st Period or Period of Units.
	Hundreds of Quadrillions. Tens of Quadrillions Quadrillions	Hundreds of Trillions Tens of Trillions Trillions	Hundreds of Billions Tens of Billions Billions	Hundreds of Millions Tens of Millions Millions	Hundreds of Thousands Tens of Thousands Thousands	Hundreds Tens Units
1	-	-	-	-	-	6
2	-	-	-	-	-	75
3	-	-	-	-	-	879
4	-	-	-	-	- 6,	023
5	-	-	-	-	- 82,	301
6	-	-	-	-	123,	087
7	-	-	-	- 7,	628,	735
8	-	-	-	- 43,	210,	460
9	-	-	-	548,	721,	087
10	-	-	- 6,	245,	289,	421
11	-	-	72,	549,	136,	822
12	-	-	894,	602,	043,	288
13	-	7,	641,	248,	907,	456
14	-	84,	912,	876,	410,	285
15	-	912,	761,	257,	327,	826
16	6,	407,	212,	936,	876,	541
17	57,	289,	678,	541,	297,	312
18	*920,*	*323,*	842,	768,	319	

---

* *Note.—This Table* is formed according to
method of numeration. The English method gi
to thousands, &c.

§ 12. The words at the head of the numeration table, units, tens, hundreds, &c. are equally applicable to all numbers, and must be committed to memory, after which, the pupil may read the Table.

To make the reading of figures easy, they are often separated into periods of three figures each, counting from the right hand.

The different lines of the table are thus read.

t line	Six.
	Seventy-five.
	Eight hundred, and seventy-nine.
4	Six thousand and twenty-three.
5	Eighty-two thousand, 3 hundred and one.
6	123 Thousand and eighty-seven.
7	7 Millions, 628 thousand, 735.
8	43 Millions, 210 thousand, 460.
9	548 Millions, 721 thousand, and 87.
10	6 Billions, 245 millions, 289 thousand, 421.
11	72 Billions, 549 millions, 136 thousand, 822.
12	894 Billions, 602 millions, 43 thousand, 288.
13	7 Trillions, 641 billions, 248 millions, 907 thousand, 456.
14	84 Trillions, 912 billions, 876 millions, 410 thousand, 285.
15	912 Trillions, 761 billions, 257 millions, 327 thousand, 826.
16	6 Quadrillions, 407 trillions, 212 billions, 936 millions, 876 thousand, 541.
17	57 Quadrillions, 289 trillions, 678 billions, 541 millions, 297 thousand, 313.
18	990 Quadrillions, 323 trillions, 842 billions, 768 millions, 319 thousand, 675.

EXAMPLES.

1. Write four, in figures. *Ans.* 4.
2. Write four tens, or forty. *Ans.* 40.
3. Write four hundred. *Ans.* 400.
4. Write four thousand. *Ans.* 4000.
5. Write forty thousand. *Ans.* 40,000.
6. Write four hundred thousand. *Ans.* 400,000.
7. Write four millions. *Ans.* 4,000,000.

These examples show us very clearly that the same significant figure will express different numbers according to the place which it occupies.

8. Write six hundred and seventy-nine. *Ans.* 679.
9. Write six thousand and twenty-one.
10. Write two thousand and forty.
11. Write one hundred and five thousand, and seven.
12. Write three billions.
13. Write ninety-five quadrillions.
14. Write one hundred and six trillions, four thousand and two.
15. Write fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousands, fifty-nine hundreds, and fifty-nine.
16. Write eleven thousand, eleven hundred and eleven.

After writing each of the examples let the pupil numerate it.

QUESTIONS.

§ 1. Of what does Arithmetic treat? When is it a science? *When an art?*

§ 2. *How are numbers* expressed? How many figures are *here? Name them.* Is there any other way of re *g numbers?* What stands for one, two, three,

§ 3. What does 0 express? What are the nine other f
ures called?

§ 4. Have we a separate character for ten? How do
express ten? To how many units 1 is it equal? May
consider it a single unit? Of what order?

§ 5. When two figures are written by the side of ea
other, what is the place on the right called? The place
the left?

§ 6. When units simply are named what units are mean

§ 7. Of what is the number 12 made up? Also, 18,
37, 54, 89, 99.

§ 8. How do you express one hundred? To how ma
units of the 2d order is it equal? To how many of the fi
order? May it be considered a single unit? Of what
der is it?

§ 9. With these combinations what numbers can be
pressed? Of what is the number three hundred and seve
ty-five composed? The number eight hundred and nine
nine?

§ 10. To what are ten units of the 3d order equal? H
do you express them? May this be considered a single un
Of what order? May any number of figures be written
a row? When so placed what do you conclude?

§ 11. What is Notation? What Numeration? Whi
way do you numerate? What is the first place called? W
the 2d? What the 3d: 4th, &c.

ADDITION OF SIMPLE NUMBERS.

§ 13. John has three apples and Charles has tw
how many apples have they between them?

Every boy will answer, five.

Here, the single apple is the unit, and the numb
five contains as many units as the two numbers thr
and two. The operation by which this result is c
tained is called *addition*.

Addition is the uniting together of several nu
bers, in such a way, that all the units which they co
may be expressed by a single number.

ngle number is called the *sum*, or sum to
er numbers. Thus 5 is the sum of
essed by John and Charles.

OF THE SIGNS.

§ 14. The sign +, is called *plus*, which signifies more. When placed between two numbers it denotes that they are to be added together. Thus, 3+2 denotes that 3 and 2 are to be added together.

The sign = is called the sign of equality. When placed between two numbers it denotes that they are equal to each other.

Thus, 3+2=5. When the numbers are small we generally read them, by saying, 3 and 2 are 5.

§ 15. Before adding large numbers the pupil should be able to add, in his mind, any two of the ten figures. Let him commit to memory the following table.

ADDITION TABLE.

2+0= 2	3+0= 3	4+0= 4	5+0= 5
2+1= 3	3+1= 4	4+1= 5	5+1= 6
2+2= 4	3+2= 5	4+2= 6	5+2= 7
2+3= 5	3+3= 6	4+3= 7	5+3= 8
2+4= 6	3+4= 7	4+4= 8	5+4= 9
2+5= 7	3+5= 8	4+5= 9	5+5=10
2+6= 8	3+6= 9	4+6=10	5+6=11
2+7= 9	3+7=10	4+7=11	5+7=12
2+8=10	3+8=11	4+8=12	5+8=13
2+9=11	3+9=12	4+9=13	5+9=14

6+0= 6	7+0= 7	8+0= 8	9+0= 9
6+1= 7	7+1= 8	8+1= 9	9+1=10
6+2= 8	7+2= 9	8+2=10	9+2=11
6+3= 9	7+3=10	8+3=11	9+3=12
6+4=10	7+4=11	8+4=12	9+4=13
6+5=11	7+5=12	8+5=13	9+5=14
6+6=12	7+6=13	8+6=14	9+6=15
6+7=13	7+7=14	8+7=15	9+7=16
6+8=14	7+8=15	8+8=16	9
6+9=15	7+9=16	8+9=17	

2+3=	how many?
1+2+4=	how many?
2+3+5+1=	how many?
6+7+2+3=	how many?
1+6+7+2+3=	how many?
1+2+3+4+5+6+7+8+9=	how many?

EXAMPLES.

Add 894 to 637.

Write the numbers thus	894 637
And draw a line beneath them	——
Then add the column of units	11
- - - - - tens	12
- - - - - hundreds	14
Sum total	1531

In this example, the sum of the units is 11, which cannot be expressed by a single figure. But 11 units are equal to 1 ten and 1 unit; therefore, we set down 1 in the place of units, and 1 in the place of tens. The sum of the tens is 12. But 12 tens are equal to 1 hundred, and 2 tens; so that 1 is set down in the hundreds place, and 2 in the tens place. The sum of the hundreds is 14. The 14 hundreds are equal to 1 thousand, and 4 hundreds; so that 4 is set down in the place of hundreds, and 1 in the place of thousands. The sum of these numbers, 1531, is the sum sought.

The example may be done in another way, thus: Having set down the numbers, as before, we say, 7 and 4 are 11: we set down 1 in the units place, and write the 1 ten under *the 3 in the col*umn of tens. We then say, *to 3 is four*, and 9 are 13. We set hree in the tens place, and write the 1 ler the 6 in the column of hundreds.

894
637
11
1531

We then add the 1, 6, and 8 together, for the hu dreds, and find the entire sum 1531, as before.

When the sum in any one of the columns excee 10, or an exact number of tens, the excess must written down, and a number equal to the number tens, added to the next left hand column.

This is called *carrying to the next column.* Th number to be carried may be written under the column or remembered and added in the mind. From these illustrations we deduce the following general

RULE.

§ 16. I. *Set down the numbers to be added so that units shall be under units, tens under tens, hundreds under hundreds, &c. and draw a line beneath them.*

II. *Begin at the foot of the units column, and add up the figures of that column. If the sum can be expressed by a single figure, write it beneath the line, in the units place. But if it cannot, see how many tens and how many units it contains. Write own the units in the units place, and carry as many o the bottom figure of the second column as there were ns in the sum. Add up that column: set down the m and carry to the third column as before.*

III. *Add each column in the same way, and set wn the entire sum of the last column.*

EXAMPLES.

. What is the sum of the numbers 375, 6321 598.

(1.)	(2.)	(3.)
375	96972	9841672
6321	3741	793139
598	9299	888923
m 7294	Sum 110012	Sum 11523734
11	2221	221111

1st, 2d and 3d examples, the small

written under the sums express the numbers to carried from one column to the other. Thus, i the 3d example, 1 is to be carried from the 1st colum to the 2d: 1 from the 2d to the 3d: 1 from t 3d to the 4th: 1 from the 4th to the 5th: 2 fro the 5th to the 6th: and 2 from the 6th to the 7th. Beginners had better set down the numbers to b carried, as in the 3d example.

PROOF OF ADDITION.

§ 17. Begin at the right hand figure of the upper line, and add all the columns downwards, carrying from one column to the other, as before. If the two results agree the work is supposed right.

SECOND PROOF.

§ 18. Draw a line under the upper number. Add the lower numbers together, and then add their sum to the upper number. If the last sum is the same as the sum total, first found, the work may be regarded as right.

QUESTIONS.

§ 13. What is addition? What is the single number called?

§ 14. What is the sign of addition? What is it called? What does it signify? When placed between two numbers what does it express? Express the sign of equality. When placed between two numbers what does it show? Give an example.

§ 16. How do you set down the numbers for addition? Where do you begin to add? When the sum of a column can be expressed by a single figure what do you do? When it cannot, what do you add to the next column? When you *add for the tens to* the next column, what is it called? What do you set down when you come to the last column? you prove addition? ou prove addition by the 2d method?

EXAMPLES.

(1.)	(2.)	(3.)
34578	22345	23456
3750	67890	78901
87	8752	23456
328	340	78901
17	350	23456
327	78	78901
Sum 39087	Sum 99755	Sum 307071
4509	77410	283615
Proof 39087	Proof 99755	Proof 307071

(4.)	(5.)	(6.)
672981043	91278976	8416785413
67126459	7654301	6915123460
39412767	876120	31810213
7891234	723456	7367985
109126	31309	654321
84172	4871	37853
72120	978	2685
787676921	100570011	15371781930

7. Add 8635, 2194, 7421, 5063, 2196 and 1245 together. *Ans.* 26754.

8. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821 and 340 together. *Ans.* 730528.

9. Add 27104, 32547, 10758, 6256, 704321, 730491, 2787316, and 2749104 together.
Ans. 7047897.

10. Add 1, 37, 29504, 6790312, 18757421 and 265 together. *Ans.* 25577540.

11. Add 562163, 21964, 56321, 18536, 4340, 279 and 83 together.
Ans. 663686.

APPLICATIONS.

1. How many days are there in the twelve
dar months? January has 31, February 28,
31, April 30, May 31, June 30, July 31, Aug
September 30, October 31, November 30, a
cember 31.

Ans.

2. A merchant on settling his accounts fin
he owes A 60 dollars, B 150 dollars, C 240 d
and to D 100 dollars. How much does he
all?

Ans. 550 *doll*

3. What is the total weight of seven ca
merchandise, viz. No. 1 weighing 960 p
No 2, 725 pounds, No. 3, 830 pounds, No.
pounds, No. 5, 698 pounds, No. 6, 569 pound
7, 987 pounds?

Ans. 5567 *pou*

4. A man borrowed a sum of money and p
part 267 dollars, and afterwards paid the rem
325 dollars: How much did he borrow?

Ans. 592 *doll*

5. At the Custom House, on the first day of
there were entered 1800 yards of linen; on the
2500 yards; on the 25th, 600 yards; on th
following, 7500 yards; and on the three las
of the month, 1325 yards each day: What w
whole amount entered during the month?

Ans. 16375 *ya*

6. A farmer has his live stock distributed
following manner: in pasture No 1, there
horses, 14 cows, 8 oxen, and 6 colts; in p
s, 4 colts, 6 cows, 20 calves a
cattle; in pasture No. 4, 320
and 5 head of young cattle

much live stock had he of each kind, and how many head had he altogether?

Ans. 8 horses, 20 cows, 8 oxen,
12 colts, 36 calves, 17 head of
young cattle, and 320 sheep.
Total live stock, 421 head.

7. What is the interval of time between a transaction which happened 125 years ago, and one that will happen 267 years hence?

Ans. 392 *years.*

8. An army consists of 4000 foot soldiers, 4006 cavalry or horse, 3093 artillery-men, 1224 riflemen, 1400 pioneers and 200 miners: What is the whole number of men in the army?

Ans. 13923.

9. The mail route from Albany to New-York is 144 miles, from New-York to Philadelphia 90 miles, from Philadelphia to Baltimore 98 miles, and from Baltimore to Washington City 38 miles. What is the distance from Albany to Washington?

Ans. 370 *miles.*

10. Suppose a man were born on the 1st of January, 1795: when will he be 85 years old?

Ans. In 1880.

11. A man dying leaves his only daughter nine hundred and ninety nine dollars, and to each of three sons two hundred dollars more than he left the daughter. What was each son's portion, and what the amount of the whole estate?

Ans. { Each son's part 1199 dolls.
Whole estate 4596 dolls.

12. What was the whole number of inhabitants in the United States in 1830; there being in Maine 399,955; New Hampshire 269,328; Vermont 280,652; Massachusetts 610,408; Rhode 97,199; Connecticut 297,675; New York 1,9

New Jersey 320,823; Pennsylvania 1,348,2 Delaware 76,748; Maryland 447,040; Virgi 1,211,405; North Carolina 737,987; South Ca na 581,185; Georgia 516,823; Alabama 309,5 Mississippi 110,357; Louisiana 215,739; Ten see 681,904; Kentucky 687,917; Ohio 937,9 Indiana 343,031; Illinois 157,445; Miss 140,455; Michigan Territory 31,639; Arka 30,388; Florida 34,730; District of Colu 39,834; Naval Service 5,602.

Ans. 12,840,54

SUBTRACTION OF SIMPLE NUMBER

§ 19. John has 6 apples and Charles has 4. many more apples has John than Charles?

Ans. 2

Two is called the *difference* between the num of apples which John has, and the number of ap which Charles has.

§ 20. *Subtraction is finding the difference bet two unequal numbers.*

§ 21. The larger of the two numbers is called *minuend*, the lesser is called the *subtrahend*, their difference is called the *remainder*.

§ 22. The sign —, is called *minus*, a term si fying less. When placed between two numbe denotes that the one on the right is to be taken f the one on the left.

Thus, 6—4=2, denotes that 4 is to be taken f 6 Here 6 is the minuend, 4 the subtrahend, an mainder.

When the numbers are small, their dif

ence is apparent, and instead of saying, 6 minus 4 equals 2, we say, 4 from 6 leaves 2. The following table is to be so read, and committed to memory.

SUBTRACTION TABLE.

2—2=0
3—2=1
4—2=2
5—2=3
6—2=4
7—2=5
8—2=6
9—2=7
10—2=8
3—3=0
4—3=1
5—3=2
6—3=3
7—3=4
8—3=5
9—3=6
10—3=7
4—4=0
5—4=1
6—4=2
7—4=3
8—4=4
9—4=5
10—4=6
5—5=0
6—5=1
7—5=2
8—5=3
9—5=4
10—5=5
6—6=0
7—6=1
8—6=2
9—6=3
10—6=4
7—7=0
8—7=1
9—7=2
10—7=3
8—8=0
9—8=1
10—8=2
9—9=0
10—9=1

12—2=10
12—3= how many?
15—4= how many?
11—6= how many?
18—9= how many?
25—8= how many?
17—7= how many?
16—8= how many?
19—9= how many?
20—4= how many?
13—7= how many?
14—2= how many?

Ex. 1. From 869 take 327.

Write the numbers { 869 minuend
327 subtrahend
542 remainder.

We begin at the right hand figure of the lower line, and say, 7 from 9 leaves 2: set down the 2 under the 7. Proceeding to the next column, we say, 2 from 6 leaves 4: set down the 4, and then ay, 3 from 8 leaves 5. Thus 542 is the remainder r true difference between the numbers.

Ex. 2. From 654 take 472.

		hundreds.	tens	units	hundreds.	tens	units
Write the	Minuend	6	5	4	5	15	4
	Subtrahend	4	7	2	4	7	2
	Remainder	1	8	2	1	8	2

Beginning at the lowest figure on the right, we say, 2 from 4 leaves 2: set down the 2. At the next step we meet a difficulty; for, we cannot subtract 7 from 5. We avoid this difficulty, thus. Ten units in the second place are equal to one unit of the third place, § 10. Therefore, if we add 10 to the 5 and diminish 6 by 1 the value of the upper line will not be changed. The numbers are so written at the right.

Now, instead of saying 7 from 5, we say, 7 from 15 leaves 8: set down the 8, and then say, 4 from 5 leaves 1.

The remainder is therefore 182. Now if instead of diminishing the 6 by 1, we had increased the 4 under it by 1, and subtracted 5 from 6, the remainder would have been the same.

§ 24. Therefore, *when a figure of the subtrahend is greater than the one directly over it, suppose* 10 *to be added to* the upper *figure. Let the lower figure be then taken from the number thus arising, and add* 1 *to the next figure of the subtrahend before it is subtracted from the figure directly above it. This is called borrowing* 10.

Ex. 3. From 6354 take 4627.

		thous.	hunds.	tens	units	thous.	hunds.	tens	units
Write the	Minuend	6	3	5	4	5	13	4	14
	Subtrahend	4	6	2	7	4	6	2	7
	Remainder.	1	7	2	7	1	7	2	7

In this example, we say, 7 from 14 leaves 7: 1 carried to 2 is 3, 3 from 5 leaves 2: 6 from 13 leaves 7: 1 carried to 4 is 5, 5 from 6 leaves 1. The remainder therefore is 1727.

Ex. 4. From 60204 take 32861.

In this example, we say, 1 from 4 leaves 3: 6 from 10 leaves 4: 1 carried to 8 is 9, 9 from 12 leaves 3: 1 carried to 2 is 3, 3 from 10 leaves 7: 1 carried to 3 is 4, 4 from 6 leaves 2.

60204	Minuend
32861	Subtrahend
27343	Remainder.

From these examples, we may deduce the following general

RULE.

§ 25. I. *Set down the less number under the greater, so that units shall fall under units, tens under tens, hundreds under hundreds, &c. and draw a line beneath them.*

II. *Then, beginning at the right hand, subtract each figure from the one directly over it, and set down the remainder.*

III. *But if the upper figure be the least, suppose it to be increased by* 10 *: then make the subtraction, set down the remainder, and carry* 1 *to the next figure of the subtrahend.*

PROOF.

§ 26. Add the remainder to the subtrahend. If their sum is equal to the minuend the work may be regarded as right.

QUESTIONS.

§ 20. *What is* Subtraction ?

§ 21. *What is the* larger of the two numbers called

the lesser called ? What is their difference called

§ 22. What is the sign of subtraction? What is it called? What does the term signify? When placed between two numbers what does it denote?

§ 23. When the numbers are small how do you read them, as 6—4?

§ 24. When a figure of the subtrahend is greater than the one above it, what do you do? What is this called?

§ 25. How do you set down the numbers for subtraction? Where do you begin to subtract? How do you subtract?

§ 26. How do you prove subtraction?

EXAMPLES.

	(1.)	(2.)	(3.)
Minuends,	8592678	67942139	219067803
Subtrahends,	1078953	9756783	104202196
Remainders,	7513725	58185356	104865607
Proofs.	8592678	67942139	219067803

	(4.)	(5.)	(6.)	(7.)	(8.)
	10000	30000	67987	100000	87000
	4	9999	40000	1	1009
Rem.	9996	20001	27987	99999	85991

9. From 2637804 take 2376982.

Ans. 260822.

10. From 3762162 take 826541.

Ans. 2935621.

11. From 78213606 take 27821890.

Ans. 50391716.

12. From thirty thousand and ninety seven, take one thousand six hundred and fifty four.

Ans. 28443.

13. From one hundred million two hundred and forty seven thousand, take one million four hundred and nine.

Ans. 99246591.

14. *Subtract* one from one million.

Ans. 999999.

APPLICATIONS.

1. Suppose John were born in eighteen hundred and fifteen, and James in eighteen hundred and twenty five: what is the difference of their ages?

Ans. 10 *years.*

2. A man was born in 1785; what was his age in 1830?

Ans. 45 *years.*

3. Suppose I lend a man 1565 dollars, and he dies, owing me 450 dollars: how much had he paid me?

Ans 1115 *dollars.*

4. In five bags are different sums of money to the amount in all of 1000 dollars. In the first there are 100 dollars; in the second, 314 dollars; in the third, 143 dollars; and in the fourth, 209 dollars; how many dollars does the fifth contain?

Ans. 234 *dollars.*

5. America was discovered by Christopher Columbus in the year 1492. What number of years has since elapsed?

6. George Washington was born in the year 1732, and died in 1799: how old was he at the time of his death?

Ans. 67 *years.*

7. The declaration of independence was published July 4th, 1776: how many years to July 4th, 1832?

Ans. 56.

8. By the census of 1830, it appeared, that the white population of the United States was 10,526,248, and the number of blacks 2,328,642. How m
did the white population exceed the black?

Ans. 8,197,

9. In 1830 there were in the State of Ne

1,918,608 inhabitants, and in the State of Pennsylvania 1,348,233 inhabitants: how many more inhabitants were there in New York than in Pennsylvania?

Ans. 570,375.

10. The revolutionary war began in 1775; the late war in 1812: what time elapsed between their commencements?

Ans. 37 *years.*

11. In 1830 there were in New York, (which is the largest city in the United States,) 207,021 inhabitants, and in Philadelphia, (the next largest city,) 161,412: how many more inhabitants were there in New York, than in Philadelphia?

Ans. 45,609.

12. A man dies worth 1200 dollars; he leaves 504 to his daughter, and the remainder to his son: what was the son's portion?

Ans. 696 *dollars.*

13. Suppose a gentleman has an income worth 3090 dollars a year, and pays for taxes 150 dollars, and expends besides 253 dollars: how much does he lay up?

Ans. 2687 *dollars.*

14. A merchant bought 500 barrels of flour for 3500 dollars; he sold 250 barrels for 2000 dollars; how many barrels remained on hand, and how much must he sell them for, that he may lose nothing?

Ans. 250 barrels remained, and he must sell for 1500 dollars.

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. A merchant buys 19576 yards of cloth of one [illegible] yards of another, and 375 yards of a

third: he sells 1050 yards to one customer, 6974 yards to another, and 10462 yards to a third: how many yards has he remaining? *Ans.* 29045.

2. A person borrowed of his neighbour at one time 355 dollars, at another time 637 dollars, and 403 dollars at another time: he then paid him 977 dollars. How much did he owe him?

Ans. 418.

3. I have a fortune of 2543 dollars to divide among my four sons, James, John, Henry, and Charles. I give James 504 dollars, John 600 dollars, and Henry 725. How much remains for Charles? *Ans.* 714 *dollars.*

MULTIPLICATION OF SIMPLE NUMBERS.

§ 27. If a number be added to itself, the sum will be *two times greater* than that number. Thus, $2+2=4$: the number 4 is 2 times greater than 2.

If 2 be repeated 3 times, we shall have $2+2+2=6$. The number 6, is *three times greater* than 2: and if 2 be written four times, the sum will be 8, a number four times greater than 2.

Any number may be repeated, as many times as we please, by adding it continually to itself.

§ 28. *Multiplication is a short method of repeating one number as many times as there are units in another.*

The number to be repeated is called the *multiplicand.*

The number denoting how many times the multiplicand is to be repeated, is called the *multiplier.*

The number arising from repeating the *multiplicand as many times* as there are units in the *multiplier, is called the product.*

§ 29. The multiplicand and multiplier are c *ctors, or producers,* of the product.

The sign ×, placed between two numbers, denot that they are to be multiplied together. It is calle the *sign of multiplication.*

MULTIPLICATION TABLE.

1 times 0 is 0	4 times 0 are 0	7 times 0 are
1 times 1 is 1	4 times 1 are 4	7 times 1 are
1 times 2 is 2	4 times 2 are 8	7 times 2 are
1 times 3 is 3	4 times 3 are 12	7 times 3 are
1 times 4 is 4	4 times 4 are 16	7 times 4 are
1 times 5 is 5	4 times 5 are 20	7 times 5 are
1 times 6 is 6	4 times 6 are 24	7 times 6 are
1 times 7 is 7	4 times 7 are 28	7 times 7 are
1 times 8 is 8	4 times 8 are 32	7 times 8 are
1 times 9 is 9	4 times 9 are 36	7 times 9 are
1 times 10 is 10	4 times 10 are 40	7 times 10 are
1 times 11 is 11	4 times 11 are 44	7 times 11 are
1 times 12 is 12	4 times 12 are 48	7 times 12 are
2 times 0 are 0	5 times 0 are 0	8 times 0 are
2 times 1 are 2	5 times 1 are 5	8 times 1 are
2 times 2 are 4	5 times 2 are 10	8 times 2 are
2 times 3 are 6	5 times 3 are 15	8 times 3 are
2 times 4 are 8	5 times 4 are 20	8 times 4 are
2 times 5 are 10	5 times 5 are 25	8 times 5 are
2 times 6 are 12	5 times 6 are 30	8 times 6 are
2 times 7 are 14	5 times 7 are 35	8 times 7 are
2 times 8 are 16	5 times 8 are 40	8 times 8 are
2 times 9 are 18	5 times 9 are 45	8 times 9 are
2 times 10 are 20	5 times 10 are 50	8 times 10 are
2 times 11 are 22	5 times 11 are 55	8 times 11 are
2 times 12 are 24	5 times 12 are 60	8 times 12 are
3 times 0 are 0	6 times 0 are 0	9 times 0 are
3 times 1 are 3	6 times 1 are 6	9 times 1 are
3 times 2 are 6	6 times 2 are 12	9 times 2 are
3 times 3 are 9	6 times 3 are 18	9 times 3 are
3 times 4 are 12	6 times 4 are 24	9 times 4 are
3 times 5 are 15	6 times 5 are 30	9 times 5 are
3 times 6 are 18	6 times 6 are 36	9 times 6 are
3 times 7 are 21	6 times 7 are 42	9 times 7 are
[illegible]es 8 are 24	6 times 8 are 48	9 times 8 are
[illegible]es 9 are 27	6 times 9 are 54	9 times 9 are
[illegible]s 10 are 30	6 times 10 are 60	9 times 10 are
[illegible] *11 are 33*	6 times 11 are 66	9 times 11 are
[illegible] *12 are 36*	6 times 12 are 72	9 times 12 are

10 times	0 are	0	11 times	0 are	0	12 times	0 are	0
10 times	1 are	10	11 times	1 are	11	12 times	1 are	12
10 times	2 are	20	11 times	2 are	22	12 times	2 are	24
10 times	3 are	30	11 times	3 are	33	12 times	3 are	36
10 times	4 are	40	11 times	4 are	44	12 times	4 are	48
10 times	5 are	50	11 times	5 are	55	12 times	5 are	60
10 times	6 are	60	11 times	6 are	66	12 times	6 are	72
10 times	7 are	70	11 times	7 are	77	12 times	7 are	84
10 times	8 are	80	11 times	8 are	88	12 times	8 are	96
10 times	9 are	90	11 times	9 are	99	12 times	9 are	108
10 times	10 are	100	11 times	10 are	110	12 times	10 are	120
10 times	11 are	110	11 times	11 are	121	12 times	11 are	132
10 times	12 are	120	11 times	12 are	132	12 times	12 are	144

Ex. 1. Let it be required to multiply 4 by 2. Here 4 is the multiplicand, and 2 is the multiplier, and it is required to find the product, which is the number arising from repeating 4 two times.

The product of 4 by 2 is found by multiplication, or by adding two 4's together.

$$\overset{\text{Multiplicand.}}{4} \times \overset{\text{Multiplier.}}{2} = \begin{array}{r} 4 \\ \underline{4} \\ 8 \end{array} \text{ Product.}$$

Ex. 2. Let it be required to multiply 4 by 3, and also to multiply 5 by 3.

$$\overset{\text{Multiplicand.}}{4} \times \overset{\text{Multiplier.}}{3} = \left\{ \begin{array}{r} 4 \\ 4 \\ \underline{4} \\ 12 \end{array} \right. \text{Product.} \qquad \overset{\text{Multiplicand.}}{5} \times \overset{\text{Multiplier.}}{3} = \left\{ \begin{array}{r} 5 \\ 5 \\ \underline{5} \\ 15 \end{array} \right. \text{Product.}$$

From these examples we see, that the product
4 multiplied by 3 is 12, the number which a
from adding three 4's together; and that the p

of 5 by 3 is equal to 15, the number which ari
from adding three 5's together.

$$\left.\overbrace{\begin{matrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{matrix}}^{5}\right\}3$$

If 3 be multiplied by 5 t
product is still 15, as befo
For, when 5 is the multip
cand and 3 the multiplier, t
product is equal to the 5 un
in the upper horizontal line taken 3 times; that
equal to 15, all the units in the 3 lines. And wh
3 is the multiplicand and 5 the multiplier, the p
duct is equal to all the units in the 5 vertical ro
which is 15.

§ 30. *Therefore, either of the factors may be us*
as the multiplier without altering the product.

We see from the above examples, that any prod
may be found by setting down the multiplicand
many times as there are units in the multiplier, a
adding all the numbers together.

§ 31. *Multiplication is therefore a short method*
addition.

$$\begin{array}{rl}236&\\ \underline{4}&\\ 24&\text{units.}\\ 12&\text{tens.}\\ \underline{8}&\text{hundred}\\ 944&\end{array}$$

Ex. 3. Multiply 236 by 4. First set down the 236, then place the 4 under the units place 6, and draw a line beneath it. Then multiply the 6 by 4: the product is 24 units; set them down. Next multiply the 3 tens by 4: the product is 12 tens; set down the 2 under the tens of the 24, leaving the
to the left, which is the place of hundreds. Ne
multiply the 2 by 4: the product is 8, which bei
hundreds, is set down under the 1. The sum
these numbers, 944, is the entire product.

The product can also be found, thus: say
4 times 6 are 24: set down the 4, and then
[illegible] 3 are 12 and 2 to carry are 14:
[illegible] and then say, 4 times 2 are
[illegible] y are 9. Set down the 9, and
as before

Ex. 4. Multiply 627 by 84.

Multiply by the 4 units as in the last example. Then multiply by the 8 tens. The first product 56 is 56 tens; the 6, therefore, must be set down under the 0, which is the place of tens, and the 5 carried to the product of the 2 by 8. Then multiply the 6 by 8, carrying the 2 from the last product, and set down the result 50. The sum of the numbers 52668, is the required product.

```
  627
   84
-----
 2508
5016
-----
52668
```

Ex. 5. Multiply 506 by 302.

In this example, we say, 2 times 6 are 12: then set down the 2, and say, 2 times 0 are 0 and 1 to carry make 1. Set down the 1, and say, 2 times 5 are 10: set down the 10. Then beginning with the 0, we say, 0 times 6 is 0: set down the 0. Then say, 0 times 0 is 0; set down the 0, and then say, 0 times 5 is 0. Then multiply by the 3 hundreds and set down the first figure 8 in the place of hundreds, and place the other figures to the left.

```
   506
   302
------
  1012
  000
1518
------
152812
```

§ 32. *When an 0 appears in the multiplier, we need not multiply by it, since each of the products is 0; but when we multiply by the next figure to the left, we must observe to set the first figure of the product directly under its multiplier.*

Thus, we have placed 8 directly under the multiplier 3.

CASE I.

§ 33. When the multiplier does not exceed 12.

RULE.

I. *Set down the multiplicand, and under it set the multiplier, so that units shall fall under units draw a line beneath them.*

II. *Multiply every figure of the multiplicand by the multiplier, setting down and carrying as in addition.*

EXAMPLES.

(1)	(2)	(3)	(4)
867901	278904	678741	3021945
1	2	3	4
867901	557808	2036223	12087780

(5)	(6)	(7)	(8)
28432	82798	6789	49604
8	9	11	12
227456	745182	74679	595248

CASE II.

§ 34. When the multiplier exceeds 12.

RULE.

I. *Set down the multiplier under the multiplicand, so that units shall fall under units, tens under tens, &c. and draw a line beneath them.*

II. *Begin with the right hand figure and multiply all the figures of the multiplicand by each figure of the multiplier, and when any of the products exceeds 9, set down and carry to the next product as in addition; observing to write the first figure of each product directly under its multiplier.*

III. *Add up the several products and their sum will be the product sought.*

PROOF OF MULTIPLICATION.

§ 35. *Write* the multiplicand in the place of the
and find the product as before: if the two
the same, the work is supposed right.

1. [illegible] times?

8. Multiply [illegible] by [illegible]

[illegible]

9. Multiply [illegible] by [illegible]

[illegible]

10. Multiply [illegible] by [illegible]

[illegible]

11. Multiply [illegible] by [illegible]

[illegible]

12. Multiply [illegible] by [illegible]

[illegible]

13. Multiply [illegible] by [illegible]

[illegible]

14. Multiply 61[illegible] by [illegible]

[illegible]

15. Multiply 718328 by 96.
Ans. 68959488.

16. Multiply 7128368 by 1440.
Ans. 10264849920.

17. Multiply 6795634 by 918546.
Ans. 6242102428164.

CASE III.

§ 36. When the multiplier is 1 and any number of ciphers after it, as 10, 100, 1000, &c.

Placing a cipher on the right of a number changes the units place into tens, the tens into hundreds, the hundreds into thousands, &c. and *therefore increases the number ten times.*

Thus, 55 is increased ten times by making it 550. So the addition of two ciphers increases a number *one hundred times;* the addition of three ciphers, *a thousand times*, &c.

Thus, 36 is increased a hundred times, by making it 3600, and 25 is increased a thousand times, by making it 25000.

Hence, we have the following

RULE.

Place as many ciphers as there are in the multiplier, on the right of the multiplicand, and the number so formed will be the required product.

EXAMPLES.

1. Multiply 254 by 10. *Ans.* 2540.
2. Multiply 648 by 100. *Ans.* 64800.
3. Multiply 7987 by 1000. *Ans.* 7987000.
4. Multiply 9840 by 10000. *Ans.* 98400000.

CASE IV.

§ 37. When there are ciphers on the right hand of one or both of the factors.

RULE.

Neglect the ciphers and multiply the significant figures: then place as many ciphers to the right hand of the product, as there are in both of the factors.

EXAMPLES.

(1.)	(2.)	(3.)
76400	7532000	416000
24	580	357000
1833600	4368560000	148512000000

4. 4871000 × 270000.
Ans. 1315170000000.

5. 296200 × 875000.
Ans. 259175000000.

6. 3456789 × 567090.
Ans. 1960310474010.

CASE V.

§ 38. When the multiplier is a composite number.

A composite number is produced by the multiplication of two or more numbers, which are called the *components* or *factors*. Thus, $2 \times 3 = 6$. Here 6 is the composite number, and 2 and 3 are the factors, or components. The number $16 = 8 \times 2$: here 16 is a composite number, and 8 and 2 are the factors; and since $4 \times 4 = 16$, we may also regard 4 and 4 as *factors or components* of 16.

Ex. 1. Let it be required to multiply 8 composite number 6, in which the factors are 2 and 3.

$$
6\left\{\begin{array}{l}
3\left\{\begin{array}{c}
\overbrace{1\;1\;1\;1\;1\;1\;1\;1}^{8}\\
1\;1\;1\;1\;1\;1\;1\;1\\
1\;1\;1\;1\;1\;1\;1\;1
\end{array}\right.\\
3\left\{\begin{array}{c}
1\;1\;1\;1\;1\;1\;1\;1\\
1\;1\;1\;1\;1\;1\;1\;1\\
1\;1\;1\;1\;1\;1\;1\;1
\end{array}\right.
\end{array}\right.
\begin{array}{l}
\left.\begin{array}{c}\\ \\ \end{array}\right\}2\times8=16\\
\left.\begin{array}{c}\\ \\ \end{array}\right\}2\\
\left.\begin{array}{c}\\ \\ \end{array}\right\}2
\end{array}
\qquad
\begin{array}{r}
16\\
3\\
\hline
48
\end{array}
\qquad
\begin{array}{r}
8\\
3\\
\hline
24\\
2\\
\hline
48
\end{array}
$$

If we write 6 horizontal lines with 8 units in each, it is evident that the product of 8×6=48, the number of units in all the lines.

But let us first connect the lines in sets of 2 each, as on the right; there will then be in each set 8×2=16; or 16 units in each set. But there are 3 sets; hence 16×3=48, the number of units in all the sets.

If we divide the lines into sets of 3 each, as on the left, the number of units in each set will be equal to 8×3=24, and there being 2 sets, 24×2=48, the whole number of units. As the same may be shown for all numbers we have the following

RULE.

When the multiplier is a composite number, multiply by each of the factors in succession, and the last product will be the entire product sought.

EXAMPLES.

1. Multiply 327 by 12.

The factors of 12 are 2 and 6, or they are 3 and 4, or they are 3, 2 and 2, for, 2×6=12, 3×4=12, an 2=12.

	327	327		327
	6	3		3
	1962	981		981
	2	4		2
Prod.	3924	3924		1962
				2
			Prod.	3924

2. Multiply 5709 by 48; the factors being 8 and 6, or 16 and 3.

Ans. 274032.

3. Multiply 342516 by 56.

Ans. 19180896.

4. Multiply 209402 by 72.

Ans. 15076944.

QUESTIONS.

§ 27. If a number be added to itself, how many times greater will the sum be than the number? How may a number be repeated?

§ 28. What is multiplication? What is the number called which is to be repeated? What is the multiplier? What is the product?

§ 29. What are the multiplier and multiplicand called? How do you denote that two numbers are to be multiplied together? What is the sign called? Repeat the table.

§ 30. If the multiplicand be made the multiplier will the product be altered?

§ 31. What may multiplication be considered?

§ 32. When an 0 appears in the multiplier, what do you do?

§ 33. When the multiplier does not exceed 12, how do you set it down? How do you multiply by it?

§ 34. When the multiplier exceeds 12, how do you set it down? How do you multiply by it? How do you add up?

§ 35. How do you prove multiplication?

§ 36. When the multiplier is 1, 10, 100, &c. what is the rule? Why?

§ 37. *When there* are ciphers on the right of both the *tors, what do you do?*

§ 38. *What is* a composite number? What are th *called? How do you* multiply by it?

APPLICATIONS.

1. There are ten bags of coffee, each containing 48 pounds: how much coffee is there in all the bags?

Ans. 480 *pounds.*

2. There are 20 pieces of cloth each containing 37 yards; and 49 other pieces, each containing 75 yards; how many yards of cloth are there in all the pieces?

Ans. 4415 *yards.*

3. There are 24 hours in a day, and 7 days in a week; how many hours in a week?

Ans. 168.

4. A merchant buys a piece of cloth containing 97 yards, at 3 dollars a yard; what does the piece cost him?

Ans. 291 *dollars.*

5. A farmer bought a farm containing 10 fields; three of the fields contained 9 acres each; three other of the fields 12 acres each; and the remaining 4 fields, each 15 acres: how many acres were there in the farm, and how much did the whole cost at 18 dollars an acre?

Ans. The farm contained
123 acres.
It cost 2214 dollars.

6. A merchant bought 49 hogsheads of molasses, each containing 63 gallons: how many gallons of molasses were there in the parcel?

Ans. 3087 *gallons.*

7. Suppose a man were to travel 32 miles a day; how far would he travel in 365 days?

Ans. 11680 *miles.*

8 a person sells goods he generally gives
bill, showing the amount charged for

them, and acknowledging the receipt of the money paid; such bills are usually called *Bills of Parcels.*

BILLS OF PARCELS.

New-York, Oct. 1, 1832.

James Johnson *Bought of W. Smith.*

4 Chests of tea, of 45 pounds each, at 1 doll. a pound.
3 Firkins of butter at 17 dolls. per firkin - -
4 Boxes of raisins at 3 dolls. per box - - -
36 Bags of coffee at 16 dolls. each - - - -
14 Hogsheads of molasses at 28 dolls. each - -

Amount 1211 dollars.

Received the amount in full.

W. Smith.

DIVISION OF SIMPLE NUMBERS.

§ 39. Having two unequal numbers, we may subtract the lesser from the greater. If the remainder be greater than the least number, that number may be subtracted from it, and these subtractions may be continued till the remainder becomes the least.

For example, take the numbers 12 and 4. Having taken 4 from 12, the remainder is 8: which, being greater than 4, 4 can be again subtracted. The second remainder is 4; and from this, 4 can be again subtracted, and the third remainder is 0. Now the number 4 has been *taken three times* from 12. T *ber 12, therefore, contains* 4 three times.

$$\begin{array}{rl} 12 & \\ \underline{4} & \\ 8 & \text{1st remain.} \\ \underline{4} & \\ 4 & \text{2d remain.} \\ \underline{4} & \\ 0 & \text{3d} \end{array}$$

§ 40. Take the numbers 27 and 8. In this example, 8 is taken 3 times from 27, and there is a remainder of 3.

$$\begin{array}{r l} 27 & \\ 8 & \\ \hline 19 & \text{1st remain.} \\ 8 & \\ \hline 11 & \text{2d remain.} \\ 8 & \\ \hline 3 & \text{3d remain.} \end{array}$$

By continued subtraction we can always find how many times one number is contained in another, and also, what is left when it is not contained an exact number of times.

We can arrive at the same result by a shorter method, called *Division.*

§ 41. Division *explains the manner of finding how many times a less number is contained in a greater. It is a short method of subtraction.*

The less number is called the *divisor*.

The greater number is called the *dividend.*

The number expressing how many times the dividend contains the divisor, is called the *quotient.*

The number which is left, when the dividend does not contain the divisor an exact number of times, is called the *remainder*.

§ 42. There are three signs used to denote division. They are the following.

$18 \div 4$ expresses that 18 is to be divided by 4.
$\frac{18}{4}$ expresses that 18 is to be divided by 4.
4)18 expresses that 18 is to be divided by 4.

When the last sign is used, a curved line is also drawn on the right of the dividend to separate it from the quotient, which is generally set down on the right.

EXAMPLES.

$\frac{16}{4}=4$ $14 \div 2=7$
$6 \div 3=2$
$=2$ $12 \div 4=3$
$9 \div 3=3$

Divisor. Dividend. Quotient.
5) 15 (3

Let the following table be committed to memory. It is read 2 in 2, 1 time; 2 in 4, 2 times, &c.

DIVISION TABLE.

2 in 2 1 time
2 in 4 2 times
2 in 6 3 times
2 in 8 4 times
2 in 10 5 times
2 in 12 6 times
2 in 14 7 times
2 in 16 8 times
2 in 18 9 times

3 in 3 1 time
3 in 6 2 times
3 in 9 3 times
3 in 12 4 times
3 in 15 5 times
3 in 18 6 times
3 in 21 7 times
3 in 24 8 times
3 in 27 9 times

4 in 4 1 time
4 in 8 2 times
4 in 12 3 times
4 in 16 4 times
4 in 20 5 times
4 in 24 6 times
4 in 28 7 times
4 in 32 8 times
4 in 36 9 times

5 in 5 1 time
5 in 10 2 times
5 in 15 3 times
5 in 20 4 times
5 in 25 5 times
5 in 30 6 times
5 in 35 7 times
5 in 40 8 times
5 in 45 9 times

6 in 6 1 time
6 in 12 2 times
6 in 18 3 times
6 in 24 4 times
6 in 30 5 times
6 in 36 6 times
6 in 42 7 times
6 in 48 8 times
6 in 54 9 times

7 in 7 1 time
7 in 14 2 times
7 in 21 3 times
7 in 28 4 times
7 in 35 5 times
7 in 42 6 times
7 in 49 7 times
7 in 56 8 times
7 in 63 9 times

8 in 8 1 time
8 in 16 2 times
8 in 24 3 times
8 in 32 4 times
8 in 40 5 times
8 in 48 6 times
8 in 56 7 times
8 in 64 8 times
8 in 72 9 times

9 in 9 1 time
9 in 18 2 times
9 in 27 3 times
9 in 36 4 times
9 in 45 5 times
9 in 54 6 times
9 in 63 7 times
9 in 72 8 times
9 in 81 9 times

10 in 10 1 time
10 in 20 2 times
10 in 30 3 times
10 in 40 4 times
10 in 50 5 times
10 in 60 6 times
10 in 70 7 times
10 in 80 8 times
10 in 90 9 times

11 in 11 1 time
11 in 22 2 times
11 in 33 3 times
11 in 44 4 times
11 in 55 5 times
11 in 66 6 times
11 in 77 7 times
11 in 88 8 times
11 in 99 9 times

12 in 12 1 time
12 in 24 2 times
12 in 36 3 times
12 in 48 4 times
12 in 60 5 times
12 in 72 6 times
12 in 84 7 times
12 in 96 8 times
12 in 108 9 times

$25 \div 5$ or $\frac{25}{5}$ =how many?
$36 \div 9$ or $\frac{36}{9}$ =how many?
$72 \div 8$ or $\frac{72}{8}$ =how many?
$54 \div 9$ or $\frac{54}{9}$ =how many?
$60 \div 5$ or $\frac{60}{5}$ =how many?
$96 \div 12$ or $\frac{96}{12}$ =how many?
$108 \div 12$ or $\frac{108}{12}$ =how many?
$99 \div 11$ or $\frac{99}{11}$ =how many?

Ex. 1. Divide 86 by 2.

Place the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.

Divisor Dividen.
2) 86
43 quotient.

Now, there are 8 tens and 6 units to be divided by 2. We say, 2 in 8, 4 times, which being 4 tens we write the 4 under the tens. We then say, 2 in 6, 3 times, which are 3 units, and must be written under the 6. The quotient therefore, is 4 tens and 3 units, or 43.

2. Divide 729 by 3.

3) 729
243

In this example there are 7 hundreds 2 tens and 9 units, all to be divided by 3. Now, we say 3 in 7, 2 times and 1 over. Set down the 2, which is hundreds, under the 7. But there is 1 hundred or 10 tens not yet divided. We put the 10 tens with the 2 tens, making 12 tens, and then say, 3 in 12, 4 times, and write the 4 in the quotient, in the tens place; then say 3 in 9, 3 times. The quotient therefore is 243.

3. Divide 729 by 9.

9) 729
81

In this example we say 9 in 7 we cannot, *but 9 in* 72, 8 times, which are 8 tens: then *9 in 9, 1 time.*

uotient is therefore 81

4. Divide 8040 by 8.

$$\begin{array}{r} 8)\underline{8040} \\ 1005 \end{array}$$

In this example, we say 8 in 8, 1 time, and set 1 in the quotient. We then say, 8 in 0, 0 times, and set the 0 in the quotient: then say, 8 in 4, 0 times, and set the 0 in the quotient: then say 8 in 40, 5 times, and set the 5 in the quotient. The true quotient is therefore 1005.

§ 43. It may be remarked that any number contains 1 as many times as there are units in the number, or that *if any number be divided by* 1, *the quotient will be equal to the number itself.*

CASE I.

§ 44. When the divisor does not exceed 12.

RULE.

I. *Set down the divisor on the left of the dividend, draw a curved line between them, and a straight line under the dividend.*

II. *Find how often the divisor is contained in the left hand figure or figures of the dividend, and place the figure so found under the straight line, for the first figure of the quotient.*

III. *If there is no remainder, divide the next figure of the dividend for the next figure of the quotient. But when there is a remainder consider it as tens, to which add the next figure of the dividend, regarded as units, and divide this sum, for the next figure of the quotient, and do the same for each of the figures of the dividend.*

IV. *When any of the figures, or sums, that are to be divided, is less than the divisor*, set down 0 in the *quotient, and to such figures* regarded as tens, add *the next figure of the* dividend considered as units, *and divide the sum for* the next figure of the quotient.

EXAMPLES.

(1)	(2)	(
8) 758904	7) 35056	6) 9
94863	5008	1

(4)
5) 364
729

In this example we find the quotient to be 7291 and a remainder 3. This 3 ought in fact to be divided by the divisor 5; but the division cannot be effected, s not contain 5. The division must then l ed by placing 5 under the 3, thus, $\frac{3}{5}$. Th tient, therefore, is $7291\frac{3}{5}$, which is read, s sand two hundred and ninety one, and *th by five.*

§ 45. Therefore, *when there is a rema division, it must be written after the qu the divisor placed under it.*

5. Divide 6794108 by 3.
Ans. 2

6. Divide 21090431 by 9.
Ans. 2

7. Divide 2345678964 by 6.
Ans. 39

8. Divide 570196382 by 12.
Ans. 475

NOTE.—This method of dividing one number l called *Short Division*.

CASE II.

§ 46. When the divisor contains seve

RULE.

I. Set down the divisor on the left of t [illegible] a [illegible] *line between them*, and [illegible] *of the dividend.*

II. *Note the fewest figures of the dividend, counted from the left hand, that will contain the divisor; find how often they contain it, and set the figure in the quotient.*

III. *Multiply the whole divisor by this figure; set the product under the first figures of the dividend, and subtract it from them. To the remainder annex the next figure of the dividend, then find how often the divisor is contained in this new number, and set the figure in the quotient.*

IV. *Multiply the whole divisor by the last figure of the quotient, and subtract the product from the last number containing the divisor. To the remainder annex the next figure of the dividend, and find the figures of the quotient in the same way, till all the figures of the dividend are brought down.*

NOTE 1. When any one of the products is greater than the number supposed to contain the divisor, the quotient figure is too large, and must be diminished.

2. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased by at least 1.

3. If after having annexed the figure from the dividend, to any one of the remainders, the number is less than the divisor, the quotient figure is 0, which being written in the quotient, annex the next figure of the dividend and divide as before.

```
Divisor.  Dividend.  Quotient.
   327)11772(36
       981
       ----
       1962
       1962
       ----
         00
```

Ex. 1. Divide 11772 by 327.

Having set down the divisor on the left of the dividend, it is seen that 327 is not contained in 117; and by observing that 3 is contained in 11, 3 times and something over, we *conclude that the divisor is contained at least 3 times* in the first four *figures of* the dividend.

Setting down the 3 in the quotient, and multiplying the divisor by it, we get 981, which being less than 1177, the quotient figure is not too great: we subtract 981 from the first four figures of the dividend, and find a remainder 196, which being less than the divisor, the quotient figure is not too small.

Annex to this remainder the next figure 2, of the dividend.

As 3 is contained in 19, 6 times, we conclude that the divisor is contained in 1962 as many as 6 times. Setting down 6 in the quotient and multiplying the divisor by it, we find the product to be 1962. Therefore the entire quotient is 36, or the divisor is contained 36 times in the dividend.

DEMONSTRATION OF THE RULE.

If 6 simple units be divided by 3, the quotient will be 2. If 6 units of the 2d order, or 60, be divided by 3, the quotient will be 2 tens, or 2 units of the 2d order. If 9 hundreds, or 9 units of the 3d order be divided by 3, the quotient will be 3 hundreds, or 3 units of the 3d order.

So, in general, *if units of any order be divided by simple units, the units of the quotient will be of the same order as those of the dividend.*

```
327)11772(36
     981
    ----
     1962
     1962
    ----
```

When in the last example it was required to divide 11772 by 327, we first considered, as we had a right to do, that 11772 is made up of 1177 tens and 2 units. We then divided the tens by the divisor 327, and found 3 tens for the quotient, by which we multiplied the divisor and subtracted the product from 1177, leaving a *remainder of* 196 tens. To this number we brought *down the 2 units*, making 1962 units. This *number contained the* divisor 6 times: that is, 6 unit's

When the unit of the first number which contains the divisor is of the 3d order, or 100, there will be 3 figures in the quotient; when it is of the 4th order there will be 4, &c.

Hence, the quotient found according to the rule, expresses the number of times which the dividend contains the divisor, and consequently is the true quotient.

PROOF OF DIVISION.

§ 47. Multiply the divisor by the quotient and add in the remainder, when there is one: the sum should be equal to the dividend.

Ex. 1. Divide 67289 by 261.

In this example we find a quotient of 257 and a remainder of 212, which being less than the divisor will not contain it.

261)67289($257\frac{212}{261}$
522
1508
1305
2039
1827
212 Rem.

PROOF.

261 Divisor
257 Quotient
1827
1305
522
212 Remainder
67289=the dividend: Hence, the work is right.

PROOF OF MULTIPLICATION.

§ 48. When two numbers are multiplied together, the multiplicand and multiplier are both factors of the product; and if the product be divided by one of the factors, the quotient will be the other factor. Hence, *if the product of two numbers be divided by the multiplicand, the quotient will be the multiplier, or, if it be divided by the multiplier, the quotient will be the multiplicand.*

EXAMPLES.

```
   3679 Multiplicand      3679)1203033(327
    327 Multiplier             11037
 -------                       -----
  25753                         9933
  7358                          7358
 11037                          -----
 -------                        25753
 1203033 Product.               25753
                                -----
                                00000
```

1. Divide 119836687 by 39407.

```
39407)119836687(3041            PROOF.
      118221           Divisor  39407
      ------           Quotient  3041
        161568                  -----
        157628                  39407
        ------                157628
          39407              118221
          39407              ---------
          -----         Div. 119836687
```

2. Divide 7210473 by 37. *Ans.* 194877$\frac{24}{37}$.

3. Divide 147735 by 45. *Ans.* 3283.

4. Divide 937387 by 54. *Ans.* 17359$\frac{1}{54}$.

45260 by 108. *Ans.* 1345.

6. Divide 79165238 by 238.

Ans. $332627\frac{12}{238}$.

7. Divide 62015735 by 7803.

Ans. $7947\frac{5224}{7803}$.

8. Divide 74855092410 by 949998.

Ans. 78795.

CONTRACTIONS IN DIVISION.

CASE I.

§ 49. When the divisor is a composite number.

RULE.

Divide the dividend by one of the factors of the divisor, and then divide the quotient thus arising by the other factor: the last quotient will be the one sought.

EXAMPLES.

Let it be required to divide 1407 dollars equally among 21 men. Here the factors of the divisor are 7 and 3.

7)1407
3)201 1st quotient
67 quot. sought

Let the 1407 dollars be first divided equally among 7 men. Each share will be 201 dollars. Let *each one* of the 7 men divide his share into 3 equal parts, each one of the three equal parts will be 67 dollars and the whole number of parts will be 21; therefore the true quotient is found by dividing continually by the factors.

2. Divide 18576 by 48=4×12.

Ans. 387.

3. Divide 9576 by 72=9×8.

Ans

4. Divide 19296 by 96=12×8.

A.

§ 50. It sometimes happens that there are remainders after division, for which we have the following

RULE.

The first remainder, if there be one, forms a part of the true remainder. The product of the second remainder, if there be one, by the first divisor, forms a second part. Either of these parts when the other does not exist, forms the true remainder, and their sum makes the true remainder when they both exist together.

EXAMPLES.

1. What is the quotient of 751 by 16?

$$4\times4=16\left\{\begin{array}{l}4\underline{)751}\\4\underline{)187}\;.\;.\;.\;.\;3\\\quad\;46\;.\;.\;.\;.\;3\times4=12\end{array}\right.$$

$$\begin{array}{r}3\\\hline 15\end{array}\text{ the true remain.}$$

Ans. $46\frac{15}{16}$.

DEMONSTRATION OF THE RULE.

In 751 there are 187 sets, (say bunches of grapes,) with 4 grapes or units in each bunch, and 3 units over. In the 187 bunches there are 46 piles, 4 bunches in a pile, and 3 bunches over. But there are 4 grapes in each bunch; therefore, the number of grapes in the 3 bunches is equal to 4×3=12, to which add 3, the grapes of the first remainder, and we have the entire remainder 15.

2. Divide 4967 by 32.

$$4\times8=32\left\{\begin{array}{l}4\underline{)4967}\\8\underline{)1241}\ldots.3\text{, 1st remainder}\\\quad\;155\ldots.1\times4+3=7\text{ true remainder.}\end{array}\right.$$

3. Divide 956789 by $7 \times 8 = 56$.

Ans. $17085\frac{29}{56}$.

4. Divide 4870029 by $8 \times 9 = 72$.

Ans. $67639\frac{21}{72}$.

5. Divide 674201 by $10 \times 11 = 110$.

Ans. $6129\frac{11}{110}$.

6. Divide 445767 by $12 \times 12 = 144$.

Ans. $3095\frac{87}{144}$.

CASE II.

§ 51. When the divisor is 10, 100, 1000, &c.

RULE.

I. *Cut off from the right hand of the dividend as many figures as there are* 0's *in the divisor.*

II. *The left hand figures of the dividend will express the quotient, and the figures cut off the remainder.*

Ex. 1. Divide 3256 by 100.

In this example there are two 0's in the divisor, therefore, there are two figures cut off from the right of the dividend, and the quotient is 32 and $56 \div 100$.

100)32|56

Ans. $32\frac{56}{100}$.

DEMONSTRATION OF THE RULE.

The quotient ought to be 10, 100, 1000, &c. times less than the dividend. But the same figure is 10, 100, 1000, &c. times greater or less in value, according to its distance from the units place, § 10. By cutting off figures from the right hand, the units place is removed to the left, and consequently the dividend is diminished 10, 100, 1000, &c. times according as you cut off 1, 2, 3, &c. figures.

2. *Divide 49763* by 10.

Ans. 4

3. Divide 7641200 by 100.

Ans. 76412.

4. Divide 496321 by 1000.

Ans. $496\frac{321}{1000}$.

CASE III.

§ 52. When there are ciphers on the right of the divisor.

RULE.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend.*

II. *Divide the remaining figures of the dividend by the significant figures of the divisor, and annex to the remainder, if there be one, the figures cut off from the dividend: this will form the true remainder.*

Ex. 1. Divide 67389 by 700.

7|00)673|89

96 . . . 1 Rem.

189 true rem.

Ans. $96\frac{189}{700}$.

In this example we strike off the 89, and then find that 7 is contained in the remaining figures, 96 times, with a remainder of 1; to this we annex 89, forming the remainder 189: to the quotient 96 we annex 189 divided by 700 for the entire quotient.

DEMONSTRATION OF THE RULE.

The number 700=100×7. Hence it is a composite number of which the factors are 100 and 7.

In striking off the two figures 89, from the right of the dividend, we divide it by 100, § 51: we then divide the 673 by the other factor 7. We then

e remainder 1 by 100 and add 89 to the

2. Divide 8749632 by 37000.

$$\begin{array}{l} 37|000)8749|632(236 \\ \quad 74 \\ \quad \overline{134} \\ \quad 111 \\ \quad \overline{\ \ 239} \\ \quad \ \ 222 \\ \quad \overline{\ \ \ \ 17} \end{array}$$

Ans. $236\frac{17632}{37000}$

3. Divide 986327 by 210000.

Ans. $4\frac{146327}{210000}$.

4. Divide 876000 by 6000.

Ans. 146.

5. Divide 36599503 by 400700.

Ans. $91\frac{135803}{400700}$.

QUESTIONS.

§ 41. What does division explain? What is the less number called? What is the greater called? What is the answer called? What is the number called which is left?

§ 42. How many signs are there in division? Make them.

§ 43. How many times does any number contain 1?

§ 44. When the divisor does not exceed 12, how do you set it down? How do you divide?

§ 45. If there is a remainder after division what do you do with it?

§ 46. When the divisor contains several figures, how do you set it down? How do you divide?

§ 47. How is division proved?

§ 48. How is multiplication proved?

§ 49. How do you divide when the divisor is a composite number?

§ 50. What do you do with the remainders after division?

§ 51. When the divisor is 1, 10, 100, &c. how do you divide?

§ 52. When there are ciphers on the right of the divisor what do you do?

APPLICATIONS IN DIVISION.

§ 53. The quotient after division shows ho ny times the dividend contains the divisor. And if we consider the dividend to be divided into as many equal parts as there are units in the divisor, the quotient will express the value of one of these parts.

Ex. 1. Divide 80 dollars equally among four men.

Here the 80 dollars is to be divided into 4 equal parts, and the quotient 20 dollars expresses the value of one of the equal parts.

$$\begin{array}{l} 4\overline{)80} \\ \overline{\;\;20} \text{ dollars.} \end{array}$$

2. Four persons buy a lottery ticket: it draws a prize of 10000 dollars. What is each one's share?
Ans. 2500 *dollars.*

3. A person dying leaves an estate of 4500 dollars to be divided equally among 5 children: what is each one's share?
Ans. 900 *dollars.*

4. There are 1560 eggs to be packed in 24 baskets: how many eggs will be put in each basket?
Ans. 65.

5. What number must be multiplied by 124 to produce 40796?
Ans. 329.

6. How many times can 24 be subtracted from 1416?
Ans. 59.

7. The sum of 19125 dollars is to be distributed among a certain number of men, each to receive 425 dollars: how many men are to receive the money?
Ans. 45.

8. By the census of 1830 the whole population of the 24 States was 12,840,540: if each one had co tained an equal number of inhabitants, how m would there have been in each state?
Ans. 535

9. If a man walks 12775 miles in a year, or 365 days, how far does he walk each day?

Ans. 35 *miles.*

10. A farmer sells a drove of sheep for 2 dollars a head, and receives 1250 dollars: how many sheep did he sell?

Ans. 625.

11. It is computed that the distance to the sun is 95,000,000 of miles, and that light is 8 minutes travelling from the sun to the earth: how many miles does it travel per minute?

Ans. 11875000.

12. By the census of 1830 it appeared that the City of New York contained 207020 inhabitants; allowing 5 to each house, how many houses were there in the city at that time?

Ans. 41404.

13. A merchant has 5100 pounds of tea and wishes to pack it in 60 chests: how many pounds must he put in each chest?

Ans. 85.

14. A person goes to a store and buys a piece of cloth containing 36 yards, for which he pays 288 dollars: how much does he pay per yard?

Ans. 8 *dollars.*

15. There are 7 days in a week: how many weeks in a year of 365?

Ans. 52 and 1 day over.

16. There are 24 hours in a day: how many days in 2040 hours?

Ans. 85.

17. Twenty three persons dined together, their bill was 92 dollars: how much had each one to pay?

Ans. 4 dollars.

GENERAL REMARKS.

§ 54. *Numeration, Addition, Subtraction,* plication and Division, are called the five ground rules of Arithmetic.

§ 55. The preceding rules furnish answers to the following questions.

Ques. 1. When the cost of each one of several things is given, how do you find their entire cost?

Ans. Add the costs of the several things together, the sum will be the entire cost.

Ques. 2. When you have two unequal numbers, how do you find their difference?

Ans. By subtracting the less from the greater.

Ques. 3. When the subtrahend and remainder are given, or known, how do you find the minuend?

Ans. By adding the remainder and subtrahend together. Hence the following principles.

> 1st. *If the sum of two numbers be diminished by one of them, the remainder will be the other number.*
>
> 2d. *The less of two numbers added to their difference, will give the greater.*

Ques. 4. When you have the cost of a single thing how will you find the entire cost of any number of things at the same rate?

Ans. Multiply the cost of the single thing by the number of things.

Ques. 5. When you know the number of things, and their entire cost, how do you find the cost of a single thing of the same kind?

Ans. Divide the entire cost by the number of things, the quotient will be the cost of a single thing.

APPLICATIONS IN THE PRECEDING RULES.

1. *A Farmer sells* a yoke of oxen for 90 dollars *ows for 25* dollars each, 9 calves for 4 dolla

each, and 65 sheep at 3 dollars a head. How much did he receive for them all?

Ans. 396 *dollars.*

2. The sum of two numbers is 365, one of the numbers is 221; what is the other number?

Ans. 144.

3. The difference of two numbers is 95, the less number is 327; what is the greater number?

Ans. 422.

4. A farmer sells 4 tons of hay at 12 dollars per ton, 80 bushels of wheat at 1 dollar per bushel, and takes in part payment a horse worth 65 dollars, a waggon worth 40 dollars, and the rest in cash. How much money did he receive?

Ans. 23 *dollars.*

5. A farmer has 14 calves worth 4 dollars each, 40 sheep worth 3 dollars each; he gives them all for a horse worth 150 dollars. Does he make or lose by the bargain?

Ans. He loses 26 dollars.

6. The product of two numbers is 51679680, and one of the factors is 615: what is the other factor?

Ans. 84032.

7. When the divisor is 67941, and the quotient 30620, what is the dividend.

Ans. 2080353420.

8. When the dividend is 1213193, the quotient 37, what is the divisor?

Ans. 32789.

9. A piece of cloth containing 65 yards cost 455 dollars; what did it cost per yard?

Ans. 7 *dollars.*

10. A man has 6 children, all of whom are m
ried, and each has four children; two of these g
children are married, and each has one child.

many children, grand-children, and great grand children are there?

Ans. 32.

11. The distance around the earth is computed to be about 25000 miles: how long would it take a man to travel it, supposing him to travel at the rate of 35 miles a day?

Ans. 714$\frac{10}{35}$ *days.*

12. The earth moves around the sun at the rate of 68 thousand miles an hour: how many miles does it travel in a day, and how many in a year?

$$\text{Ans.} \begin{cases} 1632000 \text{ in a day.} \\ 595680000 \text{ in a year.} \end{cases}$$

13. A farmer purchased a farm for which he paid 18050 dollars. He sold 50 acres for 60 dollars an acre, and the remainder stood him in 50 dollars per acre. How much land did he purchase?

Ans. 351 *acres.*

OF FRACTIONS.

§ 56. The unit 1 represents an entire thing; as 1 apple, 1 chair, 1 pound of tea.

If we suppose one thing, as one apple, or one pound of tea, to be divided into two equal parts, each part is called *one half* of the thing.

If the unit be divided into 3 equal parts, each part is called *one third.*

If the unit be divided in 4 equal parts, each is called *one fourth.*

If the unit be divided into 12 equal parts *part is called* one twelfth; and when it is d *into any* number of equal parts, we have *expression* for each of the parts.

$\frac{1}{2}$	is read	one half.
$\frac{1}{3}$	- -	one third.
$\frac{1}{4}$	- -	one fourth.
$\frac{1}{5}$	- -	one fifth.
$\frac{1}{6}$	- -	one sixth.
$\frac{1}{7}$	- -	one seventh.
$\frac{1}{8}$	- -	one eighth.
$\frac{1}{15}$	- -	one fifteenth.
$\frac{1}{50}$	- -	one fiftieth.

The $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. are called *fractions*.

§ 57. Each fraction is made up of two numbers; the number which is written above the line, is called the *numerator;* and the one below it is called the *denominator* because it gives a denomination or name to the fraction.

For example, in the fraction $\frac{1}{2}$, 1 is the numerator, and 2 the denominator. In the fraction $\frac{1}{3}$, 1 is the numerator, and 3 the denominator.

The denominator in every fraction shows into how many equal parts the unit or single thing, is divided. For example, in the fraction $\frac{1}{2}$, the unit is divided into 2 equal parts; in the fraction $\frac{1}{3}$, it is divided into 3 equal parts; in the fraction $\frac{1}{4}$, it is divided into 4 equal parts, &c. In each of the fractions *one* of the equal parts is expressed. But suppose it were required to express 2 of the equal parts, as 2 halves, 2 thirds, 2 fourths, &c.

We should then write,

$\frac{2}{2}$	they are read	two halves.
$\frac{2}{3}$	- -	two thirds.
$\frac{2}{4}$	- -	two fourths.
$\frac{2}{5}$	- -	two fifths, &c.

If it were required to express three of the equal *parts,* we should place 3 in the numerator; and generally, *the numerator shows how many of the parts are expressed in the fraction.*

For example, three eighths are written,

$\frac{3}{8}$	and read	three eighths.
$\frac{4}{9}$	- -	four ninths.
$\frac{6}{13}$	- -	six thirteenths.
$\frac{9}{20}$	- -	nine twentieths.

§ 58. When the numerator and denominator are equal, the numerator expresses all the equal parts into which the unit has been divided: therefore, the *value of the fraction is equal to* 1. But if we suppose a second unit, of the same kind, to be divided into the same number of equal parts, those parts may also be expressed in the same fraction with the parts of the first unit. Thus,

$\frac{3}{2}$	is read	three halves.
$\frac{7}{4}$	- -	seven fourths.
$\frac{16}{5}$	- -	sixteen fifths.

The denominator of the first fraction, shows that a unit has been divided into 2 equal parts, and the numerator expresses that three such parts are taken. Now, two of the parts make up one unit, and the remaining part comes from the 2d unit: hence the *value* of the fraction is $1\frac{1}{2}$; that is, one and one half.

The denominator of the second fraction shows that a unit has been divided into four equal parts, and the numerator expresses that 7 such parts are taken. Four of the 7 parts come from one unit, and the remaining 3 from a second unit: the *value* of the fraction is therefore equal to $1\frac{3}{4}$; that is, to one and three-fourths. In the third fraction, the unit has been divided into 5 equal parts, and 16 such parts [illegible] en. Now, since each unit has been divided [illegible] rts, 15 of the 16 parts make 3 units, and the [illegible] g part is 1 part of a fourth unit. There- [illegible] *value of the* fraction is $3\frac{1}{5}$: that is, three and one fifth. From what has been said, we conclude:

§ 59. 1st. *That a fraction is the expression of one or more parts of unity.*

2d. *That the denominator of a fraction shows into how many equal parts the unit or single thing has been divided, and the numerator expresses how many of such parts are taken in the fraction.*

3d. *That the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator.*

4th. *When the numerator is less than the denominator, the value of the fraction is less than* 1.

5th. *When the numerator is equal to the denominator, the value of the fraction is equal to* 1.

6th. *When the numerator is greater than the denominator, the value of the fraction is greater than* 1.

QUESTIONS.

§ 56. What does the unit 1 represent? If we divide it into two equal parts, what is each part called? If it be divided into three equal parts, what is each part? Into 4, 5, 6, &c. parts? What are such expressions called?

§ 57. Of how many parts is each fraction made up? What is the one above the line called? the one below the line? What does the denominator show? What does the numerator show?

§ 58. When the numerator and denominator are equal, what is the value of the fraction?

§ 59. Repeat the six principles.

OF FEDFRAL MONEY.

§ 60. Federal money is the currency of the United States. Its denominations, or names, are Eagles, *Dollars, Dimes,* Cents and Mills.

The coins of the United States are of gold,
and copper, and are of the following denomi

1. Gold—Eagle, half-eagle, quarter-eagle.

2. Silver—Dollar, half-dollar, quarter-dollar, dime, half-dime.

3. Copper—Cent, half-cent.

If a given quantity of gold or silver be divided into 24 equal parts, each part is called a *carat*. If any number of carats be mixed with so many equal carats of a less valuable metal, that there be 24 carats in the mixture, then the compound is said to be as many carats fine as it contains carats of the more precious metal, and to contain as much alloy as it contains carats of the baser.

For example, if 20 carats of gold be mixed with 4 of silver, the mixture is called gold of 20 carats fine, and 4 parts alloy. The standard for the gold coin is 22 carats of gold, 1 of silver and 1 of copper.

The standard for silver coins is 1485 parts of pure silver to 179 of pure copper.

The copper coins are of pure copper.

TABLE OF FEDERAL MONEY.

10 Mills marked (*m*)	make 1 Cent, marked	*ct.*
10 Cents - - -	make 1 dime, - -	*d.*
10 Dimes - - -	make 1 dollar, - -	$.
10 Dollars - -	make 1 eagle, - -	E.

§ 61. In this table 10 *units* of either denomination make one unit of the next higher denomination, and this is the same way that simple numbers increase from the right to the left, § 10. Therefore, *the denominations of federal money here expressed may be added, subtracted, multiplied and divided, by the same rules that have already been given for simple*

NUMERATION TABLE FOR FEDERAL MONEY.

Thousands of dollars.	Hundreds of dollars.	Tens of dollars or Eagles.	Dollars.	Tens of cents or dimes.	Cents.	Mills.	
					5,	7	—5 cents and 7 mills, or 57 mills.
				1	6,	4	—16 cents and 4 mills, or 164 mills.
		6	2,	1	2,	0	—62 dollars 12 cents and no mills.
	1	2	7,	6	2,	3	—127 dollars 62 cents and 3 mills.
8	9	4	0,	0	4,	1	—8940 dollars, 4 cents and 1 mill.

§ 62. As dimes are tens of cents, the second line may either be read 16 cents and 4 mills, or 1 dime 6 cents and 4 mills. And as the eagles are tens of dollars the third line may be read 62 dollars and 12 cents, or 6 eagles 2 dollars and 12 cents.

Federal money is generally read, in dollars, cents, and mills.

§ 63. From the first of the tables it appears,

1st. *That cents may be changed into mills by annexing a cipher.*

Thus, 8 cents are equal to 80 mills.

2d. *That dollars may be changed into cents by annexing two ciphers, and into mills by annexing three.*

For example, 12 dollars are equal to 1200 cents, *or to 12000* mills. The reasons of these rules *evident,* since 10 mills make a cent, 100 cent *lar, and 1000* mills a dollar.

EXAMPLES.

1. How many mills in 67 cents?

Ans. 670.

2. How many mills in $54?

Ans. 54000.

3. How many cents in $125?

Ans. 12500.

§ 64. To change mills, into dollars, cents and mills, we obtain from the numeration table the following

RULE.

Cut off the right hand figure for mills, the two next figures for cents, and the remaining left hand figures will be dollars.

The reason of the rule is this: by cutting off the first right hand figure, we in fact, divide by 10, and thus reduce the mills to cents. Then by cutting off the next two figures, we divide by 100; and thus reduce the cents to dollars.

The comma, or separatrix, is generally used to separate the mills from the cents, and the cents from the dollars. Thus, $67,25,6 is read 67 dollars 25 cents and 6 mills.

EXAMPLES.

1. How many dollars cents and mills are there in 67897 mills?

Ans. $67, 89, 7.

2. Set down 104 dollars 69 cents and 8 mills.

Ans. $104, 69, 8.

3. Set down 4069 dollars 4 cents and 2 mills.

Ans. $4069, 04, 2.

4. Set down 100 dollars 1 cent and 1 mill.

Ans. $100, 01, 1.

5. Write down 4 dollars and 6 mills.

Ans. $4, 00, 6.

6. Write down 109 dollars and 1 mill.

Ans. $109, 00, 1.

7. Write down 65 cents and 2 mills.

Ans. $0, 65, 2.

8. Write down 2 mills.

Ans. $0, 00, 2.

The parts of a dollar are sometimes expressed fractionally, as in the following

TABLE.

1 - dollar = 100 cents,
$\frac{1}{2}$ of a dollar = 50 cents,
$\frac{1}{3}$ of a dollar = $33\frac{1}{3}$ cents,
$\frac{1}{4}$ of a dollar = 25 cents,
$\frac{1}{5}$ of a dollar = 20 cents,
$\frac{1}{8}$ of a dollar = $12\frac{1}{2}$ cents,
$\frac{1}{16}$ of a dollar = $6\frac{1}{4}$ cents,
$\frac{1}{20}$ of a dollar = 5 cents,
$\frac{1}{2}$ of a cent = 5 mills.

ADDITION OF FEDERAL MONEY.

RULE.

§ 65. I. *Set down the numbers to be added under one another, so that dollars shall fall under dollars, cents under cents, and mills under mills.*

II. *Then add up the several columns as in simple numbers, and place the separating points in the amount directly under those in the columns.*

EXAMPLES.

1. Add $67, 21, 4 $10, 04, 9 $6, 04, 1 $0, 27, 1 together.

$	*cts.*	*m.*
67,	21,	4
10,	04,	9
6,	04,	1
0,	27,	1
$83,	57,	5

(2.)	(3.)	(4.)
$375, 02, 1	$ 27, 09, 8	$ 7, 00, 9
2, 09, 6	325, 59, 2	0, 01, 1
0, 47, 9	25, 60, 3	0, 00, 1
3, 01, 2	9, 99, 9	46, 67, 9
$380, 60, 8	$388, 29, 2	$53, 70, 0

APPLICATIONS.

1. A grocer purchased a box of candles for 6 dollars 89 cents; a box of cheese for 25 dollars 4 cents and 3 mills; a keg of raisins for 1 dollar 12½ cents, (or 12 cents and 5 mills;) and a cask of wine for 40 dollars 37 cents 8 mills: What did the whole cost him?

Ans. $73, 43, 6.

2. A farmer purchased a cow for which he paid 30 dollars and 4 mills; a horse for which he paid 104 dollars 60 cents and 1 mill; a wagon for which he paid 85 dollars and nine mills: how much did the whole cost?

Ans. $219, 61, 4.

3. A man is indebted to A, $630,49; to B, $25; to C, 87½ cents; to D, 4 mills: how much does he owe?

Ans. $656, 36, 9.

4. Bought 1 gallon of molasses at 28 cents per gallon; a half pound of tea for 78 cents; a piece of flannel for 12 dollars 6 cents and 3 mills; a plough for 8 dollars 1 cent and 1 mill; and a pair of shoes for 1 dollar 20 cents: What did the whole cost?

Ans. $22, 33, 4.

SUBTRACTION OF FEDERAL MONEY.

RULE.

§ 66. *Place the lesser number under the greater so that mas, or separating points, shall fall di- rec each other: then subtract as in simple* *numbers, and place the separating points in the remainder directly under those above.*

EXAMPLES.

	(1.)		(2.)
From	$204, 67, 9	From	$8976, 40, 0
Take	98, 71, 4	Take	610, 09, 8
Remainder	$105, 96, 5	Rem.	$8366, 30, 2

(3.)	(4.)	(5.)
$620, 00, 0	$327, 00, 1	$2349,
19, 02, 1	2, 09, 0	29, 33,
$600, 97, 9	$324, 91, 1	$2319, 67,

APPLICATIONS.

1. A man's income is $3000 a year; he spends $187,50: how much does he lay up?

Ans. $2812, 50.

2. A man purchased a yoke of oxen for $78, and a cow for $26, 00, 3: how much more did he pay for the oxen than for the cow?

Ans. $51, 99, 7.

3. A man buys a horse for $97,50, and gives a hundred dollar bill: how much money ought he to receive back?

Ans. $2, 50.

4. How much must be added to $60,03,9 to make the sum $1005,40?

Ans. $945, 36, 1.

5. A man sold his house for $3005, this sum being $98,03,9 more than he gave for it: what did it cost him?

Ans. $29

MULTIPLICATION OF FEDERAL MONEY

RULE.

§ 67. *Multiply as in simple numbers, and the duct will be the answer sought in the lowest den nation mentioned in the given sum; then reduce product to dollars and cents.*

Ex. 1. Multiply 375 dollars, 28 cents, and 2 m by 8.

$$\begin{array}{lr} & \$375,28,2 \\ & 8 \\ \hline \text{Product} & \$3002,25,6 \end{array}$$

(2.)	(3.)	(4.)
$3, 00, 4	$89, 07, 9	$81, 99, 2
12	7	6
$36, 04, 8	$623, 55, 3	$491, 95, 2

APPLICATIONS.

1. What will 55 yards of cloth come to at cents per yard?

Ans. $20, 35

2. What will 300 bushels of wheat come t $1,25 per bushel?

Ans. $375

3. What will 85 pounds of tea come to at 1 lar 37½ cents per pound?

In this example we first consider that ½ of a cent is equal to 5 mills. Then as 1, 37, 5 contains more figures than 85, we multiply by the 85, knowing that the product will be the same which ever number be made the mul-

70. The product 116875 which is reduced to dollars and cen

1,3

6

110

116

Ans. $116,8

4. What will a firkin of butter containing 90 pounds come to at 25½ cents per pound?

Ans. $22,95.

5. What is the cost of a cask of wine containing 29 gallons, at 2 dollars and 75 cents per gallon?

Ans. $79,75.

6. A bale of cloths contains 95 pieces costing 40 dollars 37½ cents each: what is the cost of the whole bale?

Ans. $3835,62,5.

7. What is the value of 300 hats at 3 dollars and 25 cents a piece?

Ans. $975.

8. What is the value of 9704 oranges at 3½ cents each?

Ans. $339,64.

9. What will be the cost of 356 sheep at 3¼ dollars a head?

Ans. $1157.

10. What will be the cost of 47 barrels of apples at 1¾ dollars per barrel?

Ans. $82, 25.

DIVISION OF FEDERAL MONEY.

§ 68. To divide a sum expressed in dollars, cents, and mills, by a simple number.

RULE.

I. *If the number to be divided contains dollars, cents, and mills, divide as in simple numbers, and separate the quotient into dollars, cents, and mills.*

II. *But if the number to be divided contains only dollars, or dollars and cents, bring it to mills by annexing ciphers: then divide as in simple numbe and separate the quotient as before.*

EXAMPLES.

1. Divide \$4, 62, 4 by 4; also 87, 25, 6 by 5.

4) \$4, 62, 4
\$1, 15, 6

5) \$87, 25, 6
\$17, 45, $1\frac{1}{5}$

2. Divide \$37 by 8.

8) \$37, 00, 0
\$ 4, 62, 5

In this example we first reduce the \$37 to mills by annexing three ciphers. The quotient will then be mills, and can be reduced to dollars and cents, as before.

3. Divide \$56, 17 by 16.

16)\$56, 17, 0(\$3, 51, $\frac{10}{16}$
48
81
80
17
16
10

In this example we find the quotient to be 3 dollars 51 cents, and a remainder of 10 mills, which being divided by 16 gives $\frac{10}{16}$ of a mill.

§ 69. The answer is always sufficiently exact when it is true within 1 mill, and therefore the remainder in mills may always be neglected. Indeed, in common business, the answer is sufficiently accurate, when within a cent, and the mills are generally omitted, the sign + being written after the answer to denote that the division might be still continued.

4. Divide \$495, 70, 4 by 129.

Ans. \$3, 84, +.

QUESTIONS.

§ 60. What is the currency of the United States? What are its denominations? What are the coins of the United States? Which gold, which silver, which copper? What *do you* by gold 20 carats fine? Repeat the tabl
for Fe

§ 61. How many units of either denomination make one of the next higher? How do simple numbers increase from the right to the left? How may Federal Money be added, subtracted, multiplied and divided? In numerating Federal Money what is the first figure on the right called? the second? third? fourth? &c.

§ 62. How is Federal Money generally read?

§ 63. How do you change cents into mills? How do you change dollars into cents? dollars into mills?

§ 64. How do you change mills into dollars, cents, and mills?

§ 65. How do you add Federal Money?

§ 66. How do you subtract Federal Money?

§ 67. How do you multiply Federal Money?

§ 68. How do you divide Federal Money?

§ 69. When is the answer sufficiently exact? How do you express that the division might be continued?

APPLICATIONS.

1. A man bought a piece of cloth containing 72 yards, for which he paid $252: what did he pay per yard?

Ans. $3, 50.

2. If $600 be divided equally among 26 persons, what will be each one's share?

Ans. $23, 07, +.

3. Divide $18000 into 40 equal parts: what is the value of each part?

Ans. $450.

4. Divide $3769, 25 into 50 equal parts: what is one part?

Ans. $75, 38 +.

5. A farmer purchased a farm containing 725 acres, for which he paid $18306, 25: what did it cost him per acre?

Ans. $25, 25.

6. A merchant buys 15 bales of goods at auction, *for which he pays* $1000: what do they cost him *per bale?*

Ans. $66,66

7. A drover pays $1250 for 500 sheep: what shall he sell them for a piece, that he may neither make nor lose by the bargain?

Ans. $2, 50.

8. The dairy of a farmer produces $600, and he has 25 cows: how much does he make by each cow?

Ans. $24.

9. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him?

Ans. $0, 60.

APPLICATIONS IN THE FOUR PRECEDING RULES.

1. A farmer sold a yoke of oxen for $80, 75; 6 cows for $29 each; 30 sheep at $2,50 a head; and 3 colts, one for $25, the other two for $30 a piece: what did he receive for the whole lot?

Ans. $414, 75.

2. A merchant buys 6 bales of goods, each containing 20 pieces of broadcloth, and each piece of broadcloth contained twenty-nine yards; the whole cost him $15660: how many yards of cloth did he purchase, and how much did it cost him per yard?

Ans. { 3480 *yards.*
{ $4,50 *per yard.*

3. A man dies leaving an estate of $33000 to be equally divided among his 4 children, after his wife shall have taken her third. What was the wife's portion, and what the part of each child?

Ans. { $11000 wife's part.
{ $5500 each child's part.

4. A person sells 3 cows at $25 each; and a yoke of oxen for $65; he agrees to take in payment 60 sheep: *how much* do his sheep cost him per head?

Ans. $2, 33, 3+.

rson settling with his butcher, finds that he is charged with 126 pounds of beef at 9 cents per pound; 85 pounds of veal at 6 cents per pound; 6 pairs of fowls at 37 cents a pair; and 3 hams at $1,50 each: how much does he owe him?

Ans. $23, 16.

6. A farmer agrees to furnish a merchant 40 bushels of rye at 62 cents per bushel, and to take his pay in coffee at 16 cents a pound: how much coffee will he receive?

Ans. 155 *pounds.*

7. A farmer bargains with his tailor for a new coat every six months, a new vest every three months, and three pairs of pantaloons a year: the coats to cost $29, 50 each, the vests 3 dollars a piece, and the pantaloons $12 a pair: at the end of two years how much did he owe him?

Ans. $214.

New-York, May 1st, 1832.
Mr. James Spendthrift
Bought of Benj. Saveall
16 pounds of tea at 85 cents per pound. - - - -
27 pounds of coffee at 15½ cents per pound. - - -
15 yards of linen at 66 cents per yard. - - - -
$27, 68, 5.

Rec'd payment, *Benj. Saveall.*

Albany, June 2d, 1832.
Mr. Jacob Johns *Bought of Gideon Gould*
36 pounds of sugar at 9½ cents per pound. - - -
3 hogsheads of molasses, 63 galls. each at 27 cents a gallon. - - - - - - - - -
5 casks of rice 285 pounds each, at 5 cts. per pound.
2 chests of tea 86 pounds each, at 96 cts. per
Total cost

Rec'd payment, For Gideon Gould
Charles

OF DENOMINATE NUMBERS.

§ 70. Simple numbers express a collection of units of the same kind, without expressing the particular value of the unit. For example, 40 and 55 are simple numbers, and the unit is 1, but it is not expressed whether the unit is 1 apple, 1 pound, or 1 horse.

§ 71. A denominate number expresses the *kind* of unit which is considered. For example, 6 yards of cloth is a denominate number, the *unit*, 1 yard of cloth, being denominated, or named.

§ 72. When two numbers have the same unit, they are said to be of the same denomination: and when two numbers have different units, they are said to be of different denominations.

For example, 8 feet and 10 feet are of the same denomination, the unit being 1 foot; but 30 feet and 60 yards are of different denominations, the unit of the first being 1 foot, and the unit of the second, 1 yard.

Several denominate numbers, of different denominations, are often connected together, forming a whole; as 3 feet 6 inches; 9 dollars 6 dimes and 7 cents. In the first, 1 foot is the unit of 3 feet, and 1 inch, the unit of 6 inches; and in the second, 1 dollar is the unit of 9 dollars, 1 dime the unit of 6 dimes, and 1 cent the unit of 7 cents.

The following Tables show the different kinds of denominate numbers in general use, and also their relative values.

ENGLISH MONEY.

The denominations of English Money, are *pounds*, shillings, pence, and farthings.

TABLE.

4 farthings marked *far.* make	1 penny marked	*d.*
12 pence - - -	1 shilling -	*s.*
20 shillings - - -	1 pound -	£.
21 shillings - - -	1 guinea -	

$$\begin{array}{ccccccc} \pounds & & s & & d & & far. \\ 1 & = & 20 & = & 240 & = & 960 \\ & & 1 & = & 12 & = & 48 \\ & & & & 1 & = & 4 \end{array}$$

NOTE.—Farthings are generally expressed in fractions of a penny. Thus, for 1 farthing we write $\frac{1}{4}d$, for 2 farthings, $\frac{1}{2}d$, and for 3 farthings, $\frac{3}{4}d$.

QUESTIONS.

§ 70. What are simple numbers? What is the unit? Is its value expressed?

§ 71. What is a denominate number?

§ 72. When are two numbers of the same denomination? When of different denominations? Are numbers of different denominations often connected together? Give an example.

§ 73. What are the denominations of English Money? Repeat the table. How are farthings generally expressed?

REDUCTION OF DENOMINATE NUMBERS.

§ 74. Reduction is changing the denomination of a number without altering its value.

For example, 42 dollars and 35 cents are expressed in different denominations.

But 42 dollars are equal to 4200 cents, § 63.

$$\begin{array}{rr} \text{Add} & 35 \text{ cents,} \\ \text{The sum} & \overline{4235} \text{ cents is e} \end{array}$$

42 dollars and *35* cents. Here we have bro numbers to the same denomination without altering their value.

Again, if we have 24 shillings, we can reduce them to pounds and shillings, for, since 20 shillings make 1 pound, 24 shillings are equal to £1 4s. Here we have again changed the denomination without altering the value.

§ 75. Reduction then is divided into two parts.

1st. To reduce a number from a higher denomination to a lower.

2d. To reduce a number from a lower denomination to a higher.

CASE I.

§ 76. To reduce denominate numbers from a higher denomination to a lower.

RULE.

I. *Consider how many units of the next lower denomination make one unit of the higher.*

II. *Multiply the higher denomination by that number, and add to the product the number belonging to the lower: We shall then have the equivalent number in the next lower denomination.*

III. *Proceed in a similar way through all the denominations to the last; the last sum will be the required number.*

Ex. 1. Reduce 25 Eagles 8 dollars 65 dimes and 35 cents, to the denomination of cents.

25 Eagles the highest denomination.
10 dollars make one eagle.
250 Product in dollars.
8 the number in the denomination of dolls.
258
10 the number of dimes in a dollar.
80 Product in dimes.

2580	Product in dimes.
65	the number in the denomination of dimes.
2645	
10	number of cents in a dime.
26450	Product in cents.
35	cents to be added.
26485	Number of cents in 25 Eagles, $8, 65 dimes and 35 cents.

2. Reduce £27 6*s* 8*d* to the denomination of pence.

In reducing, we often add the next lower denomination, mentally, without setting it down. In this example, when we multiply by 20, we could have added the 6*s*, without writing it down, making 6 in the product, in the units place: and when we multiplied by 12 we might have said, 12 times 6 are 72 and 8*d* to be added make 80.

The number of pence in the given denominate number is 6560.

£27 6*s* 8*d*
20
540
6*s*
546*s*
12
6552
8*d*
6560*d*

CASE II.

§ 77. To reduce denominate numbers from a lower denomination to a higher.

RULE.

I. *Consider how many units of the given denomination make one unit of the next higher; and take this number for a divisor: divide the given number by it and set down the remainder, if there be any.*

II. *Divide the quotient thus obtained by the number of units in the next higher denomination, and*
down the remainder.

Proceed in the same way through all the denominations to the highest; the last quotient with the several remainders annexed, will give the answer sought, and if there be no remainders, the last quotient will be the answer.

Ex. 1. Reduce 3138 farthings to the denomination of pounds.

4)3138
12)784 . 2 *far.* rem.
2|0)6|5 . . 4*d.* rem.
3 . . 5*s.* rem.
Ans. £3 5*s* 4*d* 2 *far.*

In this example we first divide by 4, the number of farthings in a penny: the quotient is 784 pence, and 2 farthings over. The 784 pence are then divided by 12, the number of pence in a shilling. The quotient is 65 shillings and four pence over.

The 65 shillings are then divided by 20, the number of shillings in a pound, the quotient is £3 and a remainder of 5 shillings. Hence £3 5*s* 4*d* 2 *far.* are equal to 3138 farthings.

Note.—The same rules apply to all the denominate numbers.

PROOF OF REDUCTION.

After a number has been reduced from a higher denomination to a lower, by the first rule, let it be [illegible] back by the second, and after a number [illegible] reduced from a lower denomination to a [illegible] second rule, let it be reduced back [illegible] If the results agree the work is [illegible]

[illegible] £15 7*s* 6*d* to the denomination [illegible]

	PROOF.
15	12)3690
20	20)307 . . . 6d Rem.
307	15 . . . . 7s Rem.
12	
3690	*Ans.* £15. 7s. 6d.

QUESTIONS.

§ 74. What is reduction?

§ 75. Into how many parts is reduction divided? What are they?

§ 76. How do you reduce from a higher denomination to a lower?

§ 77. How do you reduce from a lower denomination to a higher?

§ 78. How do you prove reduction?

EXAMPLES.

1. In £45 12s 10d how many pence?

Ans. 10954.

2. In 87 guineas, how many farthings?

Ans. 87696.

3. In 80 guineas how many pounds?

Ans. £84.

4. In 1549 farthings how many pounds, shillings, and pence?

Ans. £1 12s $3\frac{1}{4}$d.

5. Reduce 1046 pence to pounds.

Ans. £4 7s 2d.

6. Reduce 4704 pence to guineas.

Ans. 18 guineas, 14s.

TROY WEIGHT.

§ 79. Gold, silver, jewels, and liquors, are
ed by this weight. Its denominations are
ounces, pennyweights, and grains.

TABLE.

24 grains, *gr.* make 1 pennyweight, marked *pwt.*
20 pennyweights - 1 ounce. - - - - - *oz.*
12 ounces - - - 1 pound. - - - - - *lb.*

lb. *oz.* *pwt.* *gr.*
1=12=240=5760
1= 20= 480
1= 24

QUESTIONS.

§ 79. What things are weighed by Troy weight? What are its denominations? Repeat the Table.

EXAMPLES.

1. Reduce 16*lb.* 11*oz.* 15*pwt.* to pennyweights.

16 *lb.*
12 *oz.* in a pound.
192
11 *oz.* added.
203
20 *pwt.* in an *oz.*
4060
15 *pwt.* added.
4075 pennyweights in 16*lb.* 11*oz.* 15*pwt.*

2. In 25*lb.* 9*oz.* 0*pwt.* 20*gr.* how many grains?
Ans. 148340.

3. Reduce 6490*gr.* to pounds.

24) 6490
2|0)27|0 10*gr.* remainder.
12)13 . . 10*pwt.* remainder.
1 . . 1*oz.* remainder.
Ans. 1*lb.* 1*oz.* 10*pwt.* 10*gr*

4. In 678618 grains how many pounds?
Ans. 117*lb.* 9*oz.* 15*pwt.* 18*gr.*

[illegible]uce 8794*pwt.* to pounds.
Ans. 36*lb.* 7*oz.* 14*pwt.*

APOTHECARIES' WEIGHT.

§ 80. This weight is used by apothecaries and physicians in mixing their medicines. Its denominations are pounds, ounces, drams, scruples, and grains. The pound and ounce are the same as the pound and ounce in the Troy weight, the difference between the two weights consists in the different divisions and subdivisions of the ounce.

TABLE.

20 grains, *gr*. make 1 scruple, marked ℈.
3 scruples - - 1 dram, - - - ʒ.
8 drams - - - 1 ounce, - - - ℥.
12 ounces - - - 1 pound, - - ℔.

℔	℥	ʒ	℈	*gr*.
1=	12=	96=	288=	5760
	1=	8=	24=	480
		1=	3=	60
			1=	20

QUESTIONS.

§ 80. What is the use of the Apothecaries' weight? What are its denominations? Of what value are the pound and the ounce? Repeat the Table.

EXAMPLES.

1. Reduce 9℔ 8℥ 6ʒ 2℈ 12*gr*. to grains.

9
12
116 ounces.
8
934 drams.
3
2804 scruples.
20
Ans. 56092 grains.

2. Reduce 27℔ 9℥ 6ʒ 1℈ to scruples.
Ans. 8011 scruples.

3. Reduce 94℔ 11℥ 1ʒ to drams.
Ans. 9113 drams.

4. In 56092 grains, how many pounds?

2|0)5609|2
3)2804 12*gr.*
8) 934 2℈.
12) 116 6ʒ.
9℔ 8℥.

Ans. 9℔ 8℥ 6ʒ 2℈ 12*gr.*

5. Reduce 24033 grains to pounds.
Ans. 4℔ 2℥ 0ʒ 1℈ 13*gr.*

6. Reduce 32044 scruples to pounds.
Ans. 111℔ 3℥ 1ʒ 1℈.

AVOIRDUPOIS WEIGHT.

§ 81. By this weight are weighed all coarse articles, such as hay, grain, chandlers' wares, and all the metals, excepting gold and silver.

Its denominations are tons, hundreds, quarters, pounds, and drams.

In this weight the words *gross* and *net* are used. Gross is the weight of the goods, with the boxes, casks, or bags, in which they are contained. Net is the weight of the goods only; or what remains after deducting from the gross weight, the weight of the boxes, casks, or bags.

A hundred weight is 112 pounds, as appears from the Table. But at the present time, the merchants *in our principal* cities, buy and sell by the 100 pounds.

TABLE.

16 drams *dr.*	make	1 ounce,	marked *oz.*
16 ounces	- -	1 pound,	- - - *lb.*
28 pounds	- -	1 quarter,	- - - *qr.*
4 quarters	- -	1 hundred weight,	*cwt.*
20 hundred weight		1 ton,	- - - - T.

T	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
1=	20=	80=	2240=	35840=	573440
	1=	4=	112=	1792=	28672
		1=	28=	448=	7168
			1=	16=	256
				1=	16

QUESTIONS.

§ 81. What articles are weighed by this weight? What are its denominations? What does *gross* mean? What does *net* mean? What is a hundred weight? How do they buy and sell in the principal cities? Repeat the Table.

EXAMPLES.

1. Reduce 5 T. 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.* to drams.

5
20
108
4
435
28
12204
16
195277
16

Ans. 3124446 drams.

2. *Reduce* 27 T. 17*cwt.* 29*qr.* 21lb. to oun

Ans. 10114

3. *Reduce* 94 T. 19*cwt.* 1*qr.* to quarters.

Ans. 75

4. Reduce 3124446 drams to tons.

$$\begin{array}{rl} 4\times4=16 \left\{ \begin{array}{l} 4)3124446 \\ \quad 4)781111\ldots2 \end{array} \right. & \\ 4\times4=16 \left\{ \begin{array}{l} 4)195277\ldots3\times4+2=14dr. \\ \quad 4)48819\ldots1 \end{array} \right. & \\ 4\times7=28 \left\{ \begin{array}{l} 4)12204\ldots3\times4+1=13oz. \\ \quad 7)3051 \end{array} \right. & \\ \quad 4)435\ldots6\times4=24lb. & \\ \quad 2|0)10|8\ldots3qr. & \\ \quad\quad 5\ \ldots8cwt. & \end{array}$$

Ans. 5 T. 8*cwt.* 3*qr.* 24*lb.* 13*oz.* 14*dr.*

5. Reduce 108910592 drams to tons.

Ans. 189 T. 18*cwt.* 2*qr.*

6. Reduce 2998128 ounces to tons.

Ans. 83T. 13*cwt.* 0*qr.* 7*lb.*

LONG MEASURE.

§ 82. Long measure is used when length only is considered. Its denominations are degrees, leagues, miles, furlongs, rods, yards, feet, inches, and barley-corns.

TABLE.

3 barley-corns, *bar.* make	1 inch, marked	*in.*
12 inches - - -	1 foot, -	*ft.*
3 feet - - -	1 yard -	*yd.*
5½ yards or 16½ feet -	1 rod, perch, or pole.	*rd.*
40 rods - - -	1 furlong,	*fur.*
8 furlongs or 320 rods	1 mile, -	*mi.*
3 miles - - -	1 league, -	L.
60 geographical or 69½ statute miles	1 degree -	*deg.* or °
360 degrees - -	a great circle, or circumference of the earth.	

mi.		*fur.*		*rd.*		*yd.*		*ft.*		*in.*
1	=	8	=	320	=	1760	=	5280	=	63360
		1	=	40	=	220	=	660	=	7920
				1	=	$5\frac{1}{2}$	=	$16\frac{1}{2}$	=	198
						1	=	3	=	36
								1	=	12

Note.—A fathom is six feet, and is generally used to measure the depth of water.

A hand is 4 inches, and is used to measure the height of horses.

QUESTIONS.

§ 82. When is Long Measure used? What are its denominations? Repeat the table. What is a fathom? What is a hand?

EXAMPLES.

1. In 675*ft.* 10*in.* 2*bar.*; how many barley-corns?

```
   675
    12
  ----
  8110
     3
  ----
 24332
```

Ans. 24332 barley-corns.

2. In 59*mi.* 7*fur.* 38*rd.* how many rods?

Ans. 19198*rd.*

3. In 194656 *bar.*; how many feet?

```
 3)194656
 --------
12)64885 . . . 1bar.
   -----
    5407 . . . 1in.
```

Ans. 5407*ft.* 1*in.* 1*bar.*

4. In 115188 rods, how many miles?

Ans. 359*mi.* 7*fu*

CLOTH MEASURE.

§ 83. Cloth measure is used for me-kinds of cloth. Its denominations are Ells French, Ells English, Ells Flemish, yards, quarters, nails, and inches.

TABLE.

2¼ inches	(*in.*)	make	1 nail marked	*na.*
4 nails	-	- -	1 quarter of a yard	*qr.*
4 quarters	-	- -	1 yard - -	*yd.*
3 quarters	-	- -	1 Ell Flemish -	*E. Fl.*
5 quarters	-	- -	1 Ell English -	*E. E.*
6 quarters	-	- -	1 Ell French -	*E. Fr.*

QUESTIONS.

§ 83. For what is Cloth Measure used? What are its denominations? Repeat the table.

EXAMPLES.

1. In 35*yd.* 3*qr.* 3*na.* how many nails?

35
4
143
4
Ans. 575 *na.*

2. Reduce 49 Ells English to nails.
Ans. 980*na.*

3. Reduce 51 Ells Flemish 2*qr.* 3*na.* to nails.
Ans. 623*na.*

4. In 3278 nails, how many yards?

4)3278
4)819 . . . 2*na.*
204 . . . 3*qr.*
Ans. 204*yd.* 3*qr.* 2*na.*

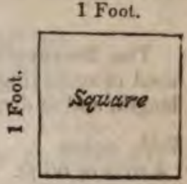
5. Reduce 340 nails to Ells Flemish.
Ans. 28 *E. Fl.* 1*qr.*

6. In 67 quarters, how many yards?
Ans. 16*yd.* 3*qr.*

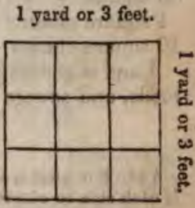
LAND OR SQUARE MEASURE.

§ 84. Land or square measure is used in measuring land, or any thing in which length and breadth are both considered.

A square is the space included between four equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be one foot, the figure is called a *square foot*.



If the sides of the square be each one yard, the square is called a *square yard*. In the large square there are nine small squares, the sides of which are each one foot. Therefore, the square yard contains 9 square feet.



The number of small squares that is contained in any large square, is always equal to the product of two of the sides of the large square. As in the figure 3×3=9 square feet. The number of square inches contained in a square foot is equal to 12×12 =144.

TABLE.

144 square inches, *sq. in.*	make	1 square foot, *Sq. ft.*
9 square feet	- -	1 square yard *Sq. yd.*
30¼ square yards	- -	1 square pole,
40 square poles	- -	1 rood, -
4 roods	- - -	1 acre, -
640 acres	- - -	1 square mil

A.	*R.*	*P.*	*Sq. yd.*	*Sq. ft.*	*Sq. in.*
1 =	4 =	160 =	4840 =	43560 =	6272640
	1 =	40 =	1210 =	10890 =	1568160
		1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39204
			1 =	9 =	1296
				1 =	144

The Surveyor's or Gunter's chain is generally used in surveying land. It is 4 poles or 66 feet in length, and is divided into 100 links.

$7\frac{92}{100}$ inches	make	1 link, marked -	*l.*
4 rods or 66 *ft.*	- - -	1 chain, - - -	*c.*
80 chains	- - - - -	1 mile, - - - -	*mi.*
1 square chain	- -	16 square poles, - -	*P.*
10 square chains	- - -	1 acre, - - - -	*A.*

Land is generally estimated in square miles, acres, roods, and square poles, or perches.

QUESTIONS.

§ 84. For what is square measure used? What is a square? If each side be one foot, what is it called? If each side be a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the table. What chain is used in surveying land? How long is it? How is it divided? Repeat the Table. How is land generally estimated?

EXAMPLES.

1. In 32M. 25A. 3R. how many square poles?

$$\begin{array}{rl}
 & 32 \\
 & 640 \\
\hline
 & 20505 \\
 & 4 \\
\hline
 & 82023 \text{ roods,} \\
 & 40 \\
\hline
\textit{Ans.} & 3280920 \text{ P.}
\end{array}$$

2. In 19A. 2R. 37P. how many square poles?
Ans. 3157*P.*

3. In 175 square chains; how many square rods?
Ans. 2800*P.*

4. In 37456 square inches, how many square feet?

$$12\times12=144\ \left\{\begin{array}{l} 12\underline{)37456} \\ 12\underline{)3121}\ .\ .\ 4 \\ \quad 260\ .\ .\ 1 \end{array}\right.$$

$1\times12+4=16.$

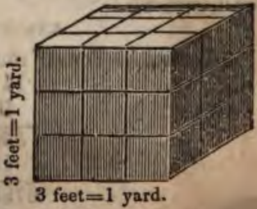
Ans. 260*Sq. ft.* 16*Sq. in.*

5. In 14972 square rods, how many acres?
Ans. 93*A.* 2*R.* 12*P.*

6. In 3674139*P.* how many square miles?
Ans. 35*M.* 563*A.* 1*R.* 19*P.*

SOLID OR CUBIC MEASURE.

§ 85. Solid or cubic measure is used in measuring stone, timber, earth, and such other things as have the three dimensions of length, breadth, and thickness. Its denominations are tons, cords, yards, feet, and inches.



A cube is a body, or solid, having six equal faces, which are squares. If the sides of the cube be each one foot long, the solid is called a cubic or solid foot. But when the sides of the cube are one yard, as in the figure, the cube is called a cubic or solid
The base of the cube, which is the face on wh
tands, contains 3×3=9 square feet. Ther
bes, of one foot each, can be placed on th

If the solid were one foot high it would contain 9 cubic feet; if it were 2 feet thick it would contain two tiers of cubes, or 18 cubic feet; and if it were 3 feet high, it would contain three tiers, or 27 cubic feet. Hence, *the content of a solid is equal to the product of its length, breadth, and thickness.* Therefore, 1 cubic foot contains 12 × 12 × 12 = 1728 cubic inches.

TABLE.

1728 solid inches, *S. in.* make	1 solid foot,	*S. ft.*
27 solid feet - -	1 solid yard	*S. yd.*
40 feet of round, or 50 feet of hewn timber,	1 ton -	Ton.
128 solid feet = 8 × 4 × 4, that is, a pile 8 feet in length, 4 feet in width, and 4 feet in height.	make 1 Cord of wood.	C.

NOTE.—A cord foot is one foot in length of the pile which makes a cord. It contains 16 solid feet.

QUESTIONS.

§ 85. For what is solid or cubic measure used? What are its denominations? What is a cube? What is a cubic or solid foot? What is a cubic yard? How many cubic feet in a cubic yard? What is the content of a solid equal to? Repeat the table. What is a cord of wood? How many solid feet does it contain?

EXAMPLES.

1. Reduce 14 Tons of round timber to solid inches.

$$\begin{array}{r} 14 \\ 40 \\ \hline 560 \\ 12 \\ \hline 6720 \\ 12 \\ \hline 80640 \\ 12 \\ \hline \end{array}$$

Ans. 967680 solid inches.

2. In 55 cords of wood, how many solid feet?
Ans. 7040.

3. In 25 cords of wood, how many cord feet?
Ans. 200 cord *ft.*

4. Reduce 3058560 cubic inches to tons of round timber.

```
1728)3058560(1770
     1728
     13305
     12096
      12096
      12096
      000000
4|0)177|0
    44 . . . 10
```

Ans. 44 tons 10*ft.*

5. Reduce 28160 solid feet to cords.
Ans. 220 cords.

6. Reduce 174964 cord feet to cords.
Ans. 21870 cords, 4 cord feet.

WINE MEASURE.

§ 86. Wine measure is used in measuring all liquors excepting beer and ale. Its denominations, are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

TABLE.

4 gills,	*gi.*	make	1 pint,	marked	*pt.*
2 pints	-	-	1 quart,	-	*qt.*
4 quarts	-	-	1 gallon	-	*g*
31½ gallons	-	-	1 barrel	-	
63 gallons	-	-	1 hogshead	-	
2 hogsheads	-	-	1 pipe	-	
2 pipes or 4 hogsheads			1 tun	-	

tun pi. hhd. bar. gal. qt. pt. gi.
1=2=4=8=252=1008=2016=8064
1=2=4=126= 504=1008=4032
1=2= 63= 252= 504=2016
1=31½= 126= 252=1008
1= 4= 8= 32
1= 2= 8
1= 4

NOTE.—A gallon, wine measure, contains 231 cubic inches.

QUESTIONS.

§ 86. What is measured by wine measure? What are its denominations? Repeat the table. What is the content of the wine gallon?

EXAMPLES.

1. In 5 tuns 1 hogshead of wine, how many gallons?

5
4
21
63
63
126
Ans. 1323 *gal.*

2. Reduce 12 pipes 1 hogshead and 1 quart of wine, to pints.

Ans. 12602*pt.*

3. In 1 tun of cider, how many gills?

Ans. 8064.

4. In 10584 quarts of wine, how many tuns?

4)10584
63=7×9 { 7)2646
9)378
4)42
Ans. 10*tuns*. 2*hhd.*

5. Reduce 201632 gills to tuns.
Ans. 25*tuns.* 1*gal.*

6. Reduce 16128 gills of cider to tuns.
Ans. 2 *tuns.*

ALE OR BEER MEASURE.

§ 87. Ale or beer measure is used in measuring ale, beer, and milk. Its denominations are hogsheads, barrels, gallons, quarts, and pints.

TABLE.

2 pints *pt.*	make	1 quart,	marked	*qt.*
4 quarts	-	1 gallon,	-	*gal.*
36 gallons	-	1 barrel,	-	*bar.*
54 gallons	-	1 hogshead,	-	*hhd.*

hhd.	*bar.*	*gal.*	*qt.*	*pt.*
1 =	$1\frac{1}{2}$ =	54 =	216 =	432
	1 =	36 =	144 =	288
		1 =	4 =	8
			1 =	2

NOTE.—A gallon, beer measure, contains 282 cubic inches.

QUESTIONS.

§ 87. For what is ale or beer measure used? What are its denominations? Repeat the Table. How many cubic inches in a gallon, beer measure?

EXAMPLES.

1. Reduce 47*bar.* 16*gal.* 4*qt.* to pints.
Ans. 13672*pt.*

2. In 27*hhd.* of beer, how many pints?
Ans. 11664.

3. In 55832 pints of beer, how many hogshead
Ans. 129hhd. 13g

4. *In 64972* quarts of beer, how many bar
Ans. 451bar.

DRY MEASURE

§ 88. Dry measure is used in measuring all dry articles, such as grain, fruits, roots, salt, coal, &c. Its denominations are chaldrons, bushels, pecks, quarts, and pints.

TABLE.

2 pints *pt.*	make	1 quart,	marked	*qt.*
8 quarts	-	1 peck,	-	*pk.*
4 pecks	-	1 bushel,	-	*bu.*
36 bushels	-	1 chaldron	-	*ch.*

ch.		*bu.*		*pk.*		*qt.*		*pt.*
1	=	36	=	144	=	1152	=	2304
		1	=	4	=	32	=	64
				1	=	8	=	16
						1	=	2

NOTE.—A gallon dry measure contains 268⅘ cubic inches. A Winchester bushel is 18½ inches in diameter, 8 inches deep, and contains 2150⅖ cubic inches.

QUESTIONS.

§ 88. What is the use of Dry measure? What are its denominations? Repeat the Table. What is the content of a gallon? How large is a Winchester bushel?

EXAMPLES.

1. In 372 bushels, how many pints?
Ans. 23808.

2. In 5 chaldrons 31 bushels, how many pecks?
Ans. 844.

3. In 17408 pints, how many bushels?
Ans. 272.

4. In 4220 pints, how many chaldrons?
Ans. 1*ch.* 29*bu.* 3*pk.* 6*qt.*

TIME.

§ 89. The denominations of time are years, months, weeks, days, hours, minutes, and seconds.

60 seconds *sec.* make	1 minute,	marked	*m.*
60 minutes - - - -	1 hour, - - - -		*hr.*
24 hours - - - - -	1 day, - - - -		*da.*
7 days - - - - -	1 week, - - - -		*wk.*
4 weeks - - - - -	1 month, - - -		*mo.*
13 mo. 1 day and 6 hrs. or 365 days, 6 hours	1 common or Julian year,	-	*yr.*

$$\begin{array}{cccccc}
yr.\ wk. & da. & hr. & m. & sec. \\
1=52\cdots & 365\frac{1}{4}= & 8766= & 525960= & 31557600 \\
1= & 7= & 168= & 10080= & 604800 \\
 & 1= & 24= & 1440= & 86400 \\
 & & 1= & 60= & 3600 \\
 & & & 1= & 60
\end{array}$$

The whole days only are reckoned. The odd six hours, by accumulating for 4 years, make one day, so that every fourth year contains 366 days. This is called the Bissextile, or Leap year.

When the Leap year occurs, the number of years up to that time is exactly divisible by 4. Thus 1800, 1804, 1808, 1812, 1816, 1820, 1824, 1828, and 1832 were all leap years.

The year is also divided into 12 calendar months, which contain an unequal number of days.

		Names.	*No. of Days.*
1	month	January, - - -	31
2	- - -	February, - - -	28
3	- - -	March, - - - -	31
4	- - -	April, - - - -	30
5	- - -	May, - - - -	31
6	- - -	June, - - - -	30
7	- - -	July, - - - -	31
8	- - -	August, - - - -	31
9	- - -	September, - -	30
10	- - -	October, - - -	31
11	- - -	November, - -	30
12	- - -	December, - - -	31
		Total -	365

The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the Leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

QUESTIONS.

§ 89. What are the denominations of Time? How long is a year? How many days in a common year? How many days in a leap year? How many calendar months in a year? Name them and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

EXAMPLES.

1. How many seconds in a year, or 365*da.* 6*hr.*?

365*da.* 6*hr.*
24
1466
730
8766 hours.
60
525960 minutes.
60
Ans. 31557600 seconds.

2. In 12 years of 365*da.* 23*hr.* 57*m.* 39*sec.* each, how many seconds?

Ans. 379467108*sec.*

3. In 49 weeks, how many minutes?

Ans. 493920.

4. In 126230400 seconds, how many years of 365 *days?*

```
          6|0)12623040|0
          6|0)210384|0
4×6=24 { 4)35064
       { 6)8766
        365)1461(4
            1460
               1
```

Ans. 4 years and 1 day.

5. In 756952018 seconds, how many years of 365 days each?

Ans. 24*yr.* 1*da.* 26*m.* 58*sec.*

6. In 5927040 minutes how many weeks?

Ans. 588.

CIRCULAR MEASURE OR MOTION.

§ 90. Circular measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies. Every circle is supposed to be divided into 360 equal parts, called degrees.

TABLE.

60 seconds	″ make	1 minute, marked		′
60 minutes	-	1 degree,	-	°.
30 degrees	-	1 sign,	-	s.
12 signs or 360°		1 circle,	-	c.

c.	s.	°.	′.	″.
1 =	12 =	360 =	21600 =	1296000
	1 =	30 =	1800 =	108000
		1 =	60 =	3600
			1 =	60

EXAMPLES.

1. *Reduce* 5s 29° 25′ to minutes.

Ans. 10765

2. In 2 circles, how many seconds?
Ans. 2592000″.

3. In 32295 minutes how many circles?
Ans. 1c. 5s. 28°. 15′.

4. In 27894 seconds, how many degrees?
Ans. 7°. 44′. 54″.

§ 91. The following denominations of things are not included in the Tables.

12 things	make	1 dozen.
12 dozen	- - -	1 gross.
12 gross, or 144 dozen	-	1 great gross.

ALSO

20 things	make	1 score.
112 pounds	- - -	1 quintal of fish.
24 sheets of paper	- -	1 quire.
20 quires	- - -	1 ream.

QUESTIONS.

§ 90. For what is circular measure used? How is every circle supposed to be divided? Repeat the Table.

§ 91. How many things make a dozen? How many dozen a gross? How many gross a great gross? How many things make a score? How many pounds a quintal of fish? How many sheets a quire of paper? How many quires a ream?

APPLICATIONS IN REDUCTION.

1. In 6169 pence, how many pounds?
Ans. £25. 14*s*. 1*d*.

2. In 59*lb.* 13*pwt.* 5*gr.*, how many grains?
Ans. 340157.

3. In £85 8*s* how many guineas?
Ans. 81 guineas 7*s*.

4. How many strokes does a regular clock strike in a year of 365 days, striking once at one, twice at two, &c. for the 24 hours in the day?
Ans. 56940.

5. In 757 boxes of sugar containing 28*lb.* each, how many tons?

Ans. 9*tons*, 9*cwt.* 1*qr.*

6. How many cords are there in a pile of wood that is 36 feet long, 6 feet high, and 4 feet wide?

Ans. 6 cords, and 6 cord feet.

7. A man has a journey to perform of 288 miles; supposing him to travel 12 hours each day for 6 days in succession, at what rate must he travel per hour to accomplish it in that time?

Ans. 4 miles.

8. How many yards of carpeting which is one yard in width, will be required to carpet a room 18 feet wide and 20 feet long?

Ans. 40 yards.

9. Reduce 346 Ells Flemish, to Ells English.

Ans. $207\frac{3}{5}$ Ells Eng.

10. Suppose the number of inhabitants in the United States to be 12 millions, how long would it take to count them, counting at the rate of 50 in a minute?

Ans. 166 days 16 hours.

11. A merchant wishes to bottle a cask of wine containing 63 gallons, into bottles containing one pint each: how many bottles are necessary?

Ans. 504.

12. There is a cube, or square piece of wood 2 feet, or 24 inches each way: how many small cubes of one inch each way can be sawed from it, allowing no waste in sawing?

Ans. 13

13. A merchant wishes to ship 285 b
flax seed, in casks containing 7 bushels 2 pec.
what number of casks are required?

An

ADDITION OF DENOMINATE NUMBERS.

§ 92. Addition of denominate numbers, like that of simple numbers, teaches how to express the value of several numbers by a single one, which is called their sum.

RULE.

I. *Set down the numbers to be added so that all the denominations of the same kind shall stand in the same column.*

II. *Begin with the column of the lowest denomination, and add it up as in whole numbers.*

III. *Then consider how many units of this denomination make one unit of the next higher, and divide the sum by this number. Write down the remainder under the units of its kind, and carry the quotient to the next column, as in addition of simple numbers.*

IV. *Proceed in the same way for all the columns to the last, and set down the entire sum of the last column.*

§ 93. The proof is the same as in the addition of simple numbers.

EXAMPLES.

ENGLISH MONEY.

	£	*s*	*d*
	4	8	9
	27	14	11
	156	17	10
Sum	£189	1*s*	6*d*
	184	12	9
Proof	189	1	6

We find the sum in the column of pence to be pence and carrying 2 to the shillings, we find 41 in the column of shillings; that is, £2 and 1 shilling. Set down 1 shilling, and carry the 2 to the pounds; their sum is 189.

§ 94. Note.—In simple numbers 10 units of any one of the columns, make one unit of the next left hand column, § 10. We therefore carry one for every 10. But in denominate numbers the higher denominations are formed differently. For example, 12 pence make 1 shilling, the unit of the next higher denomination; and 20 shillings make 1 pound. In passing from pence to shillings, we must therefore carry 1 for every 12, and one for every 20 in passing from shillings to pounds. And in general, we must carry 1 for so many units of the lower denomination, as make one unit of the next higher.

	£	*s*	*d*	£	*s*	*d*
	173	13	5	705	17	3½
	87	17	7¾	354	17	2¾
	75	18	7½	175	17	3¾
	25	17	8¼	87	19	7½
	10	10	10½	52	12	7¾
	2	5	7	27	10	5¼
Sum	376	3	10	1404	14	6½
	202	10	5	698	17	3
Proof	376	3	10	1404	14	6½

QUESTIONS.

§ 92. What is addition of denominate numbers? How do you set down the numbers for addition? Where do you begin to add? How do you add the several columns?

§ 93. How do you prove addition?

§ 94. Why do you carry one for every ten in addition of *simple numbers?* For how many do you carry one in passing *from pence to shillings?*—from shillings to pounds? In *general, for how many* do you carry one?

TROY WEIGHT.

	lb.	oz.	pwt.	gr.
Add together	11	8	18	19
	114	9	6	16
	223	10	17	12
Sum	350	5	2	23

Adding up the grains, we find their sum to be 47; that is, 1 *pwt.* and 23 *gr.*: setting down 23, and carrying 1 to the pennyweights, we find their sum to be 42: that is, 2 *oz.* and 2 *pwt.* Carrying 2 to the ounces, we find their sum to be 29; that is, 2 *lb.* and 5 *oz.*: carrying 2 to the pounds and adding, we find their sum to be 350.

	lb.	*oz.*	*pwt.*	*gr.*
Add	100	10	19	20
	432	6	0	5
	80	3	2	1
	7	0	0	9
	0	11	10	23
	0	0	8	9
Sum	621	8	1	19

lb.	*oz.*	*pwt.*	*gr.*
171	6	13	14
391	11	9	12
230	6	6	13
94	7	3	18
42	10	15	20
31	0	0	21
962	6	10	2

APOTHECARIES' WEIGHT.

℔	℥	ʒ	℈	*gr.*
24	7	2	1	16
17	11	7	2	19
36	6	5	0	7
15	9	7	1	13
9	3	4	1	9
16	10	3	2	17
4	0	1	1	12
25	2	1	0	13

℥	ʒ	℈	*gr.*
11	2	1	17
7	4	2	14
4	0	1	19
2	5	2	11
10	1	2	16
8	7	1	13
9	0	0	11
53	7	2	1

ʒ	℈	*gr.*
3	2	15
0	1	13
2	2	11
7	0	17
5	2	14
6	1	0
0	0	19
27	0	9

AVOIRDUPOIS WEIGHT.

cwt.	*qr.*	*lb.*	*oz.*	*dr.*
14	1	25	14	9
13	2	20	1	15
9	3	6	7	3
10	0	18	12	11
7	2	27	3	2
6	1	19	8	1
4	3	0	15	5
12	2	0	0	13
79	2	6	15	11

ton	*cwt.*	*qr.*	*lb.*	*oz.*
15	12	1	10	10
71	8	2	6	0
83	19	3	15	5
36	7	0	20	14
47	11	1	27	11
63	5	2	19	7
12	13	1	14	9
9	7	0	5	10
340	5	2	8	2

A merchant bought 4 barrels of potash of the following weights, viz. 1st, 3 *cwt.* 1 *qr.* 25 *lb.* 12 *oz.* 3 *dr.*; 2d. 4 *cwt.* 1 *qr.* 21 *lb.* 4 *oz.*; 3d. 4 *cwt.*; 4th. 3 *cwt.* 3 *qr.* 27 *lb.* 15 *oz.* 15 *dr.*: What was the entire weight of the four barrels?

Ans. 15 *cwt.* 3 *qr.* 19 *lb.* 0 *oz.* 2 *dr.*

LONG MEASURE.

L.	*mi.*	*fur.*	*rd.*
16	2	7	39
327	1	2	20
87	0	1	15
1	1	1	1
432	2	4	35

yd.	*ft.*	*in.*	*bar.*
90	2	11	2
155	1	9	1
327	0	7	0
50	2	1	2
624	1	5	2

CLOTH MEASURE.

E. Fl.	*qr.*	*na.*
126	4	4
65	3	1
72	1	3
157	2	2
424	0	2

yd.	*qr.*	*na.*
4	3	2
5	4	1
6	1	0
25	2	2
42	3	1

E. E.	*qr.*	*na.*
128	5	1
20	3	1
19	1	4
15	3	
184	3	

LAND OR SQUARE MEASURE.

Sq. yd.	*Sq. ft.*	*Sq. in.*	*M.*	*A.*	*R.*
97	4	104	1	700	3
22	3	27	6	375	2
105	8	2	7	450	1
37	7	127	11	30	0
263	5	116	27	277	0

There are 4 fields, the first contains 12 A 38P.; the 2d. 4A. 1R. 26P.; the 3d. 85A 19P.; and the 4th. 57A. 1R. 2P.: how many in the four fields?

Ans. 159A. 2R.

SOLID OR CUBIC MEASURE.

S. yd.	*S. ft.*	*S. in.*	*C.*	*S. ft.*	*C. Cor*
65	25	1129	16	127	87
37	26	132	17	12	26
50	1	1064	18	119	16
22	19	17	37	104	19
176	18	614	90	106	151

WINE MEASURE.

hhd.	*gal.*	*qt.*	*pt.*	*tun*	*pi.*	*hhd.*	*gal*
127	65	3	2	14	2	1	27
12	60	2	3	15	1	2	25
450	29	0	1	4	2	1	27
21	0	2	3	5	0	1	62
14	39	1	2	7	1	2	21
627	7	1	1	50	0	1	38

DRY MEASURE.

ch.	*bu.*	*pk.*	*qt.*	*pt.*
27	25	3	7	1
59	21	2	6	3
2	1	2	7	1
5	9	1	8	2
94	22	3	7	1

ch.	*bu.*	*pk.*	*qt.*	*pt.*
141	36	3	7	2
21	32	2	4	1
85	9	1	0	3
10	4	4	1	3
259	12	0	0	1

TIME.

yr.	*mo.*	*wk.*	*da.*	*hr.*
4	11	3	6	20
3	10	2	5	21
5	8	1	4	19
101	9	3	7	23
55	8	4	6	17
172	2	1	4	4

wk.	*da.*	*hr.*	*m.*	*sec.*
8	8	14	55	57
10	7	23	57	49
20	6	14	42	01
6	5	23	19	59
2	2	20	45	48
50	4	1	41	34

SUBTRACTION OF DENOMINATE NUMBERS.

§ **95.** Subtraction of denominate numbers teaches how to find the difference between two denominate numbers that are unequal.

RULE.

I. *Set down the lesser number under the greater, placing the same denominations directly under each other.*

II. *Begin with the lowest denomination, and if the number expressing that denomination be less than the number directly over it, make the subtraction as in simple numbers. But if it* be greater, subtract it *from the upper number* increased by so many units *make one unit of the next* higher denomination, *carry this one which has been borrowed to the next higher denomination, as in subtraction of simple numbers.*

III. *Do the same for all the denominations, and set down the several remainders, and they will form the true remainder.*

EXAMPLES.

	£.	s.	d.	£.	s.	d.
From	27	16	8	26	35	20
Take	19	17	9	19	17	9
Remainder	7	18	11	7	18	11

In this example we cannot take 9*d.* from 8*d.*; we therefore add 12*d.* to the 8*d.* making 20*d.*, and then say, 9 from 20, 11 remains. Set down the 11, and carry one to 17 making 18: then say 18 from 36 leaves 18: set it down and carry one to 19 making 20: 20 from 27 leaves 7.

Or we may set down the minuend as on the right: £26. 35*s.* 20*d.* Then 9 from 20 leaves 11; 17 from 35 leaves 18; and 19 from 26, 7 remains.

	T.	*cwt.*	*qr.*	*lb.*
From	4	12	3	20
Take	2	18	2	26
Remainder	1	14	0	22

In this example, we take 26 from 48; then 3 from 3; then 18 from 32; then 3 from 4.

PROOF.

§ 96. Add the remainder to the subtrahend—their sum should be equal to the minuend.

QUESTIONS.

§ 95. **What is subtraction of denominate numbers? How write down the numbers for subtraction? Where do to subtract? When the number to be subtracte**

is less than the one above it, what do you do? When it is greater, what do you do?

§ 96. How do you prove subtraction?

EXAMPLES.

	A.	R.	P.
From	18	3	28
Take	15	2	30
Remainder	3	0	38
Proof	18	3	28

	lb.	*oz.*	*pwt.*	*gr.*	*lb.*	*oz.*	*pwt.*	*gr.*
From	273	0	0	0	18	9	10	8
Take	98	10	18	21	9	10	15	20
Rem.	174	2	1	3	8	10	14	12

	T.	*cwt.*	*qr.*	*lb.*	*oz.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
From	7	14	1	3	6	14	2	12	10	8
Take	2	6	3	4	11	6	3	16	15	3
Rem.	5	7	1	26	11	7	2	23	11	5

APPLICATIONS IN ADDITION AND SUBTRACTION.

1. Sold a merchant one quarter of beef for £2 7*s* 9*d*; one cheese for 9*s* 7*d*; 20 bushels of corn for £4 10*s* 11*d*; and 40 bushels of wheat for £19 12*s* 8½*d*: how much did the whole come to?

Ans. £27 11½*d.*

2. Bought of a silversmith a tea pot, weighing 3*lb.* 4*oz.* 9*pwt.* 21*gr.*; one dozen of silver spoons, weighing 2*lb.* 1*oz.* 1*pwt.*; 2 dishes weighing 16*lb.* 10*oz.* 15*pwt.* 16*gr.*: how much did the whole weigh?

Ans. 22*lb.* 4*oz.* 6*pwt.* 13*gr.*

3. Bought one hogshead of sugar, weighing 9*cwt.* 2*qr.* 27*lb.* 14*oz.*; one barrel weighing 3*cwt.* 27*lb.*, and a second barrel weighing 2cwt. 3qr. 26lb. 4oz.: *how much did* the whole weigh?

Ans. 15cwt. 3qr. 25lb.

4. A merchant buys two hogsheads of sugar,
weighing 8*cwt.* 3*qr.* 21*lb.*; the other, 9*cwt.* 2*qr.* 6
he sells two barrels, one weighing 3*cwt.* 1*qr.* 1
14*oz.*; the other, 2*cwt.* 3*qr.* 15*lb.* 6*oz.*: how m
remains on hand?

Ans. 12*cwt.* 26*lb.* 12*oz*

5. A man sets out upon a journey and has
miles to travel; the first day he travels 9 league
miles 7 furlongs 30 rods; the second day 12 leag
1 mile 1 furlong; the third day 14 leagues;
fourth day 15 leagues 2 miles 5 furlongs 35 ro
how far had he then to travel?

Ans. 14L. 1*mi.* 1*fur.* 15*rd*

6. A farmer has two meadows, one contain
9A. 3R. 37P., the other containing 10A. 2R. 25
also three pastures, the first containing 12A.
1P., the second containing 13A. 3R., and the th
6A. 1R. 39P.: by how many acres does the p
ture exceed the meadow land?

Ans. 11A. 3R. 18P

7. Supposing the declaration of independenc
have been published at precisely 12 o'clock on
4th of July 1776, how much time elapsed to the
of January 1833, at 25 minutes past 3 P. M.?

Ans. 56*yr.* 181*da.* 3*hr.* 25*m*

8. A farmer has three granaries, one for wh
one for rye, and one for corn: he fills them all.
wheat granary contains 657*bu.* 3*pk.* 6*qt.*; the c
granary 257*bu.* 1*pk.* 1*qt.*; the rye granary 459
2*pk.* 7*qt.*: how much grain had he in all, and h
much more wheat than rye?

Ans. { In all 1374*bu.* 3*pk.* 6*qt.*
{ Wheat more than rye 198*bu.* 1*q*

9. A father was born on the 8th of Dece

1759, his first son on the 4th of June, 1795: what was the difference of their ages?

Ans. 35*yr.* 5*mo.* 27*da.*

10. A merchant has a bill to pay of £600. He has £250 19*s* 8*d* in cash, a good note against A for £75 10*s* 6*d* and a note against B for £37 11*s* 9*d*: how much money does he want to make the payment?

Ans. £235 18*s* 1*d.*

11. A tailor requires 1*yd.* 3*qr.* 3*na.* of cloth for a father's coat and 1*yd.* 1*qr.* 2*na.* for each of the two sons: the father buys 6 yards, does he buy too much or too little?

Ans. 1*yd.* 1*qr.* 1*na.* too much.

MULTIPLICATION OF DENOMINATE NUMBERS.

§97. To multiply a denominate number by a simple one is to repeat the denominate number as many times as there are units in the multiplier.

CASE I.

§98. When the simple number does not exceed 12.

RULE.

I. *Write down the denominate number and set the multiplier under the lowest denomination.*

II. *Multiply the lowest denomination by the multiplier, and see how many units of the next higher denomination are contained in the product, and set down the excess as in addition.*

III. *Multiply the next higher* denomination by the *multiplier and add the units* to be carried from *last product; then* reduce the sum to units *next higher denomination*, write down the ex *proceed in the same way for all the denominations, setting down the entire product when you come to the last.*

QUESTIONS.

§ 97. What is required when you are to multiply a denominate number by a simple one?

§ 98. When the simple number does not exceed 12, how do you write it down? How do you begin to multiply? How do you carry?

EXAMPLES.

Multiply £3 9*s* 10*d* by 4.

In this example we say, 4 times 10*d* are 40*d*, equal to 3*s* and 4*d*. Set down the 4*d* in the lower line. Then 4 times 9*s* are 36*s* and 3*s* to carry make 39*s*, equal to £1 and 19*s* over: set down the 19*s*. Then 4 times £3 are £12 and £1 to carry make £13.

£	*s*	*d*
3	9	10
		4
£12	36*s*	40*d*
£13	19*s*	4*d*

(2.)

£	*s*	*d*
17	15	9
		6
106	14	6

(3.)

T.	*cwt.*	*qr.*	*lb.*	*oz.*
	9	3	27	12
				7
3	9	3	26	4

APPLICATIONS.

1. What is the cost of 4 yards of cloth at £1 3*s* 6*d* per yard?

The amount per yard multiplied by the number of yards will evidently give the entire cost.

£1	3*s*	6*d*
		4
£4	14*s*	0*d* *Ans.*

2. What will be the cost of 9 hats, at 9*s* 9*d* each?
Ans. £4 7*s* 9*d*.

3. A farmer has 11 bags of corn each containing 2*bu.* 1*pk.* 3*qt.*: how much corn in all the bags?

Ans. 25*bu.* 3*pk.* 1*qt.*

4. What is the cost of 12 bushels of wheat at 9*s.* 6*d.* per bushel?

Ans. £5 14*s.*

5. How much sugar in 12 barrels, each containing 3*cwt.* 2*qr.* 27*lb.*?

Ans. 2*T.* 4*cwt.* 3*qr.* 16*lb.*

6. In 7 loads of wood, each containing 1 cord and 2 cord feet, how many cords?

Ans. 8 cords. 6 cord feet.

CASE II.

§ 99. When the simple number is greater than 12 and a composite number.

RULE.

Multiply the denominate number by one of the component parts, or factors, and then multiply the product by the other factors in succession: the last product is the one required.

Ex. 1. Multiply £6 2*s* 9*d* by 48=6×8.

Ans. £294 12*s.*

2. What will 24 barrels of flour cost, at £2 11*s* 8*d* per barrel?

Ans. £62.

3. What is the cost of 42*cwt.* of tallow, at £1 14*s* 6*d* per cwt.?

Ans. £72 9*s.*

4. What is the cost of 120 dozen of candles at 5*s* 9*d* per dozen?

Ans. £34 10*s.*

5. How much water will be contained in 9
heads, each containing 62*gal.* 1*qt.* 1*pt.* 1*gi*

Ans. 5991

CASE III.

§ 100. When the simple number exceeds 1
is not a composite number.

RULE.

Multiply the simple number by each of the d
nations separately, and reduce each product
highest denomination named. Then add the s
products together, and their sum will be the a
sought.

Ex. 1. Multiply £5 3*s* 8*d* by 13.

13	13	1
8*d*	3*s*	£
104*d*=8*s* 8*d*	39*s*=£1 19*s*	£6

£65
1 19*s*
8*s* 8*d*
Ans. £67 7*s* 8*d*.

2. Multiply £6 8*s* 9*d* by 139.

139 × 9*d*=1251*d*=£ 5 4*s* 3*d*
139 × 8*s* =1112*s* =£ 55 12*s*
139 × 6£=£834 =£834 00
Ans. £894 16*s* 3*d*.

3. Multiply £0 2*s* 4*d* by 195.
Ans. £22

4. What is the cost of 46 bushels of whea
7¼*d* per bushel? *Ans.* £10 11*s*

5. What is the cost of 117*cwt.* of raisins at
3*d* per cwt.?
Ans. £130 3*s*

QUESTIONS.

§ 99. *How do you multiply when the simple nu*
greater than 12 and a composite number?

§ 100. *How do you multiply when the simple nu*
ceeds 12 and is not a composite number?

DIVISION OF DENOMINATE NUMBERS.

§ 101. A denominate number may be divided into any number of equal parts by dividing each of its denominations by the divisor.

RULE.

I. *Set down the number to be divided in the order of its denominations from the highest to the lowest, and write the divisor on the left.*

II. *Find how often the divisor is contained in the figures of the highest denomination.*

III. *Reduce the remainder, if there be any, to the next lower denomination, and add the figures of the dividend expressing that denomination, and then divide the sum by the divisor.*

IV. *Proceed in the same way for all the denominations, to the last, and if there be a remainder place the divisor under it, as in division of simple numbers. Each of the quotients will be of the same denomination as its dividend, and the several quotients connected together will be the entire quotient sought.*

PROOF OF MULTIPLICATION.

§ 102. Divide the product by the multiplier, and if the quotient is equal to the multiplicand, the work may be considered right.

PROOF OF DIVISION

§ 103. Multiply the quotient by the divisor, and if the product is equal to the dividend, the work may be considered right.

QUESTIONS.

§ 101. How may a denominate number be d
do you set down the number to be divided?

then divide? When there is a remainder what do you do with it? Of what denomination will each of the quotients be?

§ 102. How do you prove multiplication?

§ 103. How do you prove division?

Ex. 1. Divide £25 15*s* 10*d* into 8 equal parts.

In this example we find that 8 is contained in £25, 3 times and £1 over. Now this £1 has yet to be divided by 8, as well as the 15*s* and 10*d*. Therefore by multiplying the £1 by 20 and adding the 15*s* gives 35*s*, which contains 8, 4 times and 3*s* over. Multiplying the 3*s* by 12 and adding in the 10*d*, gives 46*d*, which contains 8, 5 times and 6*d* over. The 6*d* being reduced, gives 24 farthings which contains 8, 3 times. Therefore each of the denominations has been divided by 8, and the reason of the rule is plain.

8)£25 15*s* 10*d*(£3
24
£1
20
8)35*s*(4*s*
32
3*s*
12
8)46(5*d*
40
6*d*
4
8)24*far*.(3*far*.

Ans. £3 4*s* $5\frac{3}{4}$*d*.

2. Divide 36*bu*. 3*pk*. 7*qt*. by 7.

In this example we find that 7 is contained in 36 bushels 5 times and 1 bushel over. Reducing this to pecks, and adding 3 pecks gives 7 pecks, which contains 7, 1 time and no remainder. Multipying 0 by *8 quarts* and adding, gives *7 quarts to* be divided by 7.

7)26*bu*. 3*pk*. 7*qt*.(5*bu*.
35
1
4
7)7*pk*.(1*pk*.
7
0
8
7)7(1qt.

Ans. 5bu. 1pk. 1qt.

NOTE.—When the divisor does not exceed 12 the division may be made after the manner of short division in simple numbers.

Thus in the first example, we say 8 into 25, 3 times and £1 or 20*s* over. Then after adding the 15*s*, we say, 8 into 35, 4 times and 3*s* over. Then reducing the 3*s* to pence and adding in the 4*d*, we say 8 into 40, 5 times.

$$\begin{array}{r|rrr} 8 & \pounds 25 & 15s & 4d \\ \hline & \pounds\ 3 & 4s & 5d \end{array}$$

3. Divide £821 17*s* 9$\frac{3}{4}$*d* by 4.
Ans. £205 9*s* 5*d* 1$\frac{3}{4}$*far*.

4. Divide £55 14*s* $\frac{3}{4}$*d* by 7.
Ans. £7 19*s* 1*d* 3$\frac{2}{7}$*far*.

5. Divide 16*cwt*. 3*qr*. 27*lb*. 6*oz*. by 7.
Ans. 2*cwt*. 1*qr*. 19*lb*. 14$\frac{4}{7}$*oz*.

6. Divide 49*yd*. 3*qr*. 3*na*. by 9.
Ans. 5*yd*. 2*qr*. $\frac{7}{9}$*na*.

7. Divide 131*A*. 1*R*. by 12.
Ans. 10*A*. 3*R*. 30*P*.

8. Divide £1138 12*s* 4*d* by 53.
Ans. £21 9*s* 8*d*.

9. Divide 1417*cwt*. 7*lb*. by 79.
Ans. 17*cwt*. 3*qr*. 21*lb*.

10. Divide £23 15*s* 7$\frac{1}{2}$*d* by 37.
Ans. 12*s* 10$\frac{1}{4}$*d*.

11. Divide £199 3*s* 10*d* by 53.
Ans. £3 15*s* 2*d*.

NOTE 2.—When the divisor is a composite number, and exceeds 12, the work may be s
dividing by the factors, in succession, a
of simple numbers.

Ex. 1. Divide £28 2s 4d by the co
ber 21. Here the factors are 3 and 7.

7)£28 2*s* 4*d*	3)£4 0*s* 4*d*
£4 0*s* 4*d*	£1 6*s* 9⅓*d*

Hence the answer sought is £1 6*s* 9⅓*d*.

2. Divide £57 3*s* 4*d* by 35=5×7.

Ans. £1 12*s* 8*d*.

3. Divide £85 4*s* by 72.

Ans. £1 3*s* 8*d*.

4. Divide £31 2*s* 10½*d* by 99.

Ans. 6*s* 3½*d*.

APPLICATIONS.

1. Bought 65 yards of cloth for which I paid £72 14*s* 4½*d*: what did it cost per yard?

Ans. £1 2*s* 4½*d*.

2. Bought 64 gallons of brandy for £30 8*s*: what did it cost per gallon?

Ans. 9*s* 6*d*.

3. Bought 144 reams of paper for £96: what did it cost me per ream?

Ans. 13*s* 4*d*.

4. Sixty three barrels of sugar contain 7*T*. 16*cwt*. 3*qr*. 21*lb*: how much is there in each barrel?

Ans. 2*cwt*. 1*qr*. 27*lb*.

5. A farmer has a granary containing 232 bushels 3 pecks 7 quarts of wheat, and he wishes to put it in 105 bags: how much will each bag contain?

Ans. 2*bu*. 7*qt*.

6. One hundred and seventy six men consumed in a week 13*cwt*. 3*qr*. 1*lb*. 6*oz*. of bread: how much did each man consume?

Ans. 8*lb*. 12*oz*. 2*dr*.

APPLICATIONS IN THE FOUR RULES.

Albany, July 1, 1833.
Mr. James Sears
Bought of Albert Titus.

3*lb.* of green tea at 7*s* 6*d* per pound. - - -
27*yd.* of muslin at 1*s* 6*d* per yard. - - -
4*cwt.* of sugar at £2 2*s* 8*d* per cwt. - - -
2*hhd.* of molasses at 2*s* 6*d* per gallon. - -
6*lb.* of raisins at 1*s* 7*d* per pound. - - -

Received payment, £27 18*s* 2*d.*
Albert Titus.

2. A gentleman purchased of a silversmith, 2 dozen silver spoons each weighing 3*oz.* 4*pwt.* 1*gr.*; 2 dozen of tea spoons, each weighing 15*pwt.* 16*gr.*; 3 tankards each weighing 22*oz.* 14*pwt.*; he sold him old silver to the amount of 6*lb.* 10*oz.* 3*pwt.*: How much remained to be paid for?

Ans. 6*lb.* 9*oz.* 12*pwt.*

3. What will be the cost of 22 tons of hay, at £2 1*s* 10*d* per ton?

Ans. $46 0*s* 4*d.*

4. If two hogsheads of wine cost £67 4*s*; what does it cost per gallon?

Ans. 10*s* 8*d.*

5. If 4*cwt.* of sugar cost £14; what is it per pound?

Ans. 7½*d.*

6. A man paid £67 4*s* for a pile of wood containing 64 cords: he sold 30 cords for £29 16*s*: for how much must he sell the remainder per cord so as not to lose?

Ans. £1 2*s*.

7. If 78*cwt.* 3*qr.* 10*lb.* of sugar be equally divi
among 5 men, what will be each one's share?

Ans. 15*cwt.* 3*qr.*

8. A printer uses one sheet of paper for ev pages of an octavo book: how much paper necessary to print 500 copies of a book con 336 pages, allowing 2 quires of waste paper i ream?

Ans. 24 reams 5 quires 12 sh

9. A farmer wishes to divide 108 acres equal fields: how much will there be in each *Ans.* 13A.

10. Out of a pipe of wine, a merchant dra bottles, each containing 1 pint 3 gills; he th six 5 gallon demijohns; then he draws off 3 bottles, each containing 1 quart 2 gills: how remained in the cask?

Ans. 82*gal.*

11. A man lends his neighbour £135 6*s* takes in part payment 4 cows at £5 8*s* a piec a horse worth £50: how much remained du *Ans.* £63 14*s*

12. A farmer has 6*T.* 8*cwt.* 2*qr.* 14*lb.* of be removed in 6 equal loads: how much m carried at each load?

Ans. 1*T.* 1*cwt.* 1*qr.* 2

13. A person at his death left landed estate amount of £2000, and personal property amount of £2803 17*s* 4*d.* He directed that his should receive one eighth of the whole, an the residue should be equally divided among h children. What was the widow's and each portion?

Ans. { Widow's portion £600 9*s* 8*d*
{ Each child's portion £1050 16*s*

2 4/0,

RULE OF THREE.

§ 104. *Ex.* 1. If 1 yard of cloth cost $2, how much will 6 yards cost at the same rate?

It is plain that 6 yards of cloth, at the same rate will cost 6 times as much as 1 yard, and therefore, the whole cost is found by multiplying $2 by 6—giving $12 for the cost. In this example there are four numbers considered, viz. 1 yard of cloth, 6 yards of cloth, $2 and $12: these numbers are called *terms*. Three of these terms were known or given in the question, and the other was to be found.

1 yard of cloth is the 1st term;
6 yards of cloth is the 2d term;
$2 is the - - - - 3d term; and
$12 is the - - - - 4th term.

Now the 2d term 6 contains the first term 1, 6 times, and the 4th term 12 contains the 3d term 2, 6 times—that is, *the 2d term is as many times greater than the* 1st, *as the 4th term is greater than the* 3d.

§ 105. This relation between four numbers is called *geometrical proportion:* and generally

Four numbers are in geometrical proportion, when the 2d term is as many times greater or less than the 1st, *as the 4th term is greater or less than the* 3d.

We express that four numbers are in proportion thus:

1 : 6 : : 2 : 12

That is, we write the numbers in the same line and place two dots between the 1st and 2d terms, four between the 2d and 3d terms, and two between the 3d and 4th terms. We read the proportion thus,

as 1 is to 6, so is 2 to 12.

The *numbers* 2 : 4 : : 5 : 10 are in geor *proportion, since* the 2d term is twice the *the 4th term* twice the 3d.

The 1st and 2d terms of a proportion always express quantities of the same kind, and so likewise do the 3d and 4th terms. As in the first example.

	yd.		*yd.*		$		$
	1	:	6	: :	2	:	12

Ex. 2. If 4*lb.* of tea cost $8, what will 12*lb.* cost at the same rate?

	lb.		*lb.*		$		$	$
As	4	:	12	: :	8	:	answer	8
					12			3
					4)96		*Ans.*	$24
					$24 the cost of 12*lb* of tea.			

It is evident that the 4th term, or cost of 12*lb* of tea, must be as many times greater than $8, the cost of 4*lb*, as 12*lb.* is greater than 4*lb.* But since the quotient of 12 divided by 4 expresses how many times 12 is greater than 4, it follows that the fourth term will be equal to $8 multiplied by this quotient: that is, equal to $8 multiplied by 3, or equal to $24. But we obtain the same result whether we multiply the 3d term $8 by the quotient 3, or first multiply it by the 2d term and then divide the product by the 1st term; and the same may be shown for every proportion. Hence, we conclude,

§ 106. *That the fourth term of every geometrical proportion may be found by multiplying the 2d and 3d terms together, and dividing their product by the first term.*

§ 107. *The 1st and 4th terms of a proportion are called the two extremes, and the 2d and 3d terms are called the two means.* Now, since the 4th term is obtained by dividing the product of the 2d and 3d by the first term, and since the product of the divisor by the quotient is equal to the dividend, it follows,

That in any geometrical proportion the product of the two extremes is equal to the product of the two means.

Thus in the first example,

1 : 6 : : 2 : 12 we have, $1\times12=6\times2=12$
and in the second,
4 : 12 : : 8 : 24 $4\times24=8\times12=96$.

§ 108. The Rule of Three takes its name from the circumstance that, three numbers are always given to find a fourth, which shall bear the same proportion to one of the given numbers as exists between the other two.

GENERAL RULE.

§ 109. I. *Reduce the two numbers which have different names from the answer sought, to the lowest denomination named in either of them.*

II. *Set the number which is of the same kind with the answer sought in the third place, and then consider from the nature of the question whether the answer will be greater or less than the third term.*

III. *When the answer is greater than the third term, write the least of the remaining numbers in the first place, but when it is less, place the greater there.*

Then multiply the second and third terms together and divide the product by the first term: the quotient will be the fourth term or answer sought, and will be of the same denomination as the third term.

Ex. 3. If 48 yards of cloth cost $67,25, what will 144 yards cost at the same rate?

yd. yd. $ $
48 : 144 :: 67,25 : answer
144
26900
26900
6725
48)9684,00($201,75
96
84
48
360
336
240
240
000

In this example, as the answer is to be dolla
place the $67,25 in the third term. Then, s
yards of cloth will cost more than 48 yards, the
term must be greater than the third, and the
we write the least of the two remaining numb
the first place. The product of the 2d and 3d
is $9684,00: then dividing by the first term w
tain $201,75 for the cost of 144 yards of cloth

Ex. 4. If 6 men can dig a certain ditch in 4(
how many days would 30 men be employed i
ging it?

men men days days
30 : 6 :: 40 : answer
6
3|0)24|0
Ans. 8 days.

As the answer must be days, the 40 days in
hird place. Then as it is evident tha

will do the same work in a shorter time than six men, it is plain that the fourth term must be less than the third: therefore 30 men, the greater of the remaining numbers, is written in the first term. Besides, it is plain that the fourth term must be just so many times less than 40, as 6 is less than 30.

Ex. 5. If 25 yards of cloth cost £2 3*s* 4*d*, what will 5 yards cost at the same rate?

yd. *yd.* £ *s* *d*
25 : 5 : : 2 3 4 : answer
5
25)£10 16*s* 8*d*
20
25)216(8*s*
200
16
12
25)200(8*d*
200

When we come to divide the product of the 2d and 3d terms by the first, it is found the £10 does not contain 25. We then reduce to the next lower denomination and divide as in division of denominate numbers.

Ex. 6. If 3*cwt.* of sugar cost £9 2*s* 0*d*, what will 4*cwt.* 3*qr.* 26*lb.* cost at the same rate?

3*cwt.* 4*cwt.* 3*qr.* 26*lb.*
4 4 £9 2*s* 0*d*
12 19 20
7 7 182*s*
4×7=28 { 84 133 12
4 4 2184
336*lb.* : 558*lb* : : 2184d : ans

336lb. : 558lb. : : 2184d : answer

558

17472
10920
10920

336)1218672(3627d÷12
1008

2106
2016

907
672

2352
2352

2|0)30|2s . . 3d
£15 . . . 2s

Ans. £15 2s 3d.

PROOF.

§ 110. If the product of the two mean terms divided by either of them, the quotient will be other. But the product of the two extremes is e to the product of the two means § 107, therefore have the following

RULE.

Divide the product of the first and fourth by the second term, and if the work is right th tient will be equal to the third term.

QUESTIONS.

§ 104. If 1 yard of cloth cost $2, what will 6 ya How many numbers are considered in this questio are they called? How many were known, or give the terms. How many times does the 2d term co first? How many times does the fourth contain How many times is the 2d term greater than the § 105. When are four numbers in geometrical are they written? What terms of a propo of the same kind?

§ 106. How do you find the 4th term of a proportion?

§ 107. What are the first and fourth terms called? What are the second and third terms called? What is the product of the two extremes equal to?

§ 108. From what does the Rule of Three take its name?

§ 109. What is the first thing to be done in stating a question? Which number do you make the third term? How do you determine which to put in the first? After stating the question, how do you find the 4th term? What will be its denomination?

§ 110. How do you prove the Rule of Three?

APPLICATIONS.

1. If 4 hats cost $12, what will 55 cost at the same rate?

Ans. $165.

2. What is the value of 2*cwt*. of sugar at 5*d*. per pound?

Ans. £4 13*s* 4*d*.

3. If 40 yards of cloth cost $170, what will 325 yards cost?

Ans. $1381,25.

4. If 240 sheep yield 660*lb*. of wool, how many pounds will be obtained from 1200?

Ans. 3300*lb*.

5. If 2 gallons of molasses cost 65 cents, what will 3 hogsheads cost?

Ans. $61,42½.

6. If a man travels at the rate of 210 miles in 6 days, how far will he travel in a year supposing him not to travel on Sundays?

Ans. 10955 miles.

7. If 1 yard of cloth cost $3,25, what will be the cost of 3 pieces each containing 25 yards?

Ans. $243,75

8. If 30 barrels of flour will support 100 men 40 days, how long would it subsist 25 men?

Ans. 160 da

9. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 400 men?

Ans. 10 days.

10. A owes B £679 6*s*., but compounds with him by paying 3*s*. 4*d*. on the pound: how much does B receive of his debt?

Ans. £113 4*s*. 4*d*.

11. If 90 bushels of oats will feed 40 horses for 6 days, how long would 450 bushels last them?

Ans. 30 days.

12. If 5*cwt*. 3*qr*. 14*lb*. of sugar cost £6 1*s*. 8*d*., what will 35*cwt*. 28*lb*. cost?

Ans. £36 10*s*.

13. What is the cost of 3*cwt*. of coffee at 15*d*. per pound?

Ans. £21.

14. If 3 quarters of a yard of velvet cost 7*s*. 3*d*., how many yards can be bought for £13 15*s*. 6*d*.?

Ans. 28*yd*. 2*qr*.

15. If an ingot of gold weighing 9*lb*. 9*oz*. 12*pwt*. be worth £470 8*s*., what is that per grain?

Ans. 2*d*.

16. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards, at £16 4*s*. per piece: what is the value of the whole, and the cost per yard?

Ans. £388, 16*s*. at 12*s*. per yard.

17. What will be the cost of 72 yards of cloth, at the rate of £5 12*s*. for 9 yards?

Ans. £44 16*s*.

18. A person's annual income is £146: how much is that per day?

Ans. 8*s*.

19. If 3 paces, or common steps of a person, be *equal to* 2 yards, how many yards will 160 paces

Ans. 106yd. 2ft.

20. What length must be cut off from a board that is 9 inches wide, to make a square foot, that is, as much as is contained in 12 inches in length and 12 in breadth?

Ans. 16 inches.

21. If 750 men require 22500 rations of bread for a month, how many rations will a garrison of 1200 men require?

Ans. 36000.

22. If 7*cwt.* 1*qr.* of sugar cost $64,96, what will be the price of 4*cwt.* 2*qr.*?

Ans. $40,32.

23. The clothing of a regiment of foot of 750 men amounts to £2831 5*s.*: what will it cost to clothe a body of 3500 men?

Ans. £13212 10*s.*

24. How many yards of carpeting, that is 3 feet wide, will cover a floor that is 27 feet long and 20 feet broad?

Ans. 60 yards.

25. What is the cost of 6 bushels of coal at the rate of £1 14*s.* 6*d.* the chaldron?

Ans. 5*s.* 9*d.*

26. If 6352 stones of 3 feet long will complete a certain quantity of wall, how many stones of 2 feet long will raise the like quantity?

Ans. 9528.

27. If a person can count 300 in 2 minutes, how many can he count in a day?

Ans. 216000.

28. A garrison of 536 men have provisions for 365 days: how long will those provisions last if the garrison be increased to 1124 men?

Ans. 174 days and

29. What will be the tax upon £763 *rate of 3s. 6d.* per pound sterling?

Ans. £133

30. What will be the tax on $3758, at the rate of 4 mills on the dollar?

Ans. $15,03,2.

31. A certain work can be raised in 12 days by working 4 hours each day: how long would it require to raise the work by working 6 hours per day?

Ans. 8 days.

32. What quantity of corn can I buy for 90 guineas, at the rate of 6 shillings a bushel?

Ans. 315 bushels.

33. A person failing in trade owes £977, at which time he has in money, goods, and recoverable debts £420 6*s*. $3\frac{1}{2}$*d*.: now supposing an equal division among his creditors how much will they get on the pound?

Ans. 8*s*. $7\frac{1}{4}$*d*.

34. A pasture of a certain extent having supplied a body of horse, consisting of 3000, with forage for 18 days, how many days would the same pasture have supplied a body of 2000 horse?

Ans. 27 days.

35. Suppose a gentleman's income to be 600 guineas a year, and that he spends 25*s*. 6*d*. per day, one day with another; how much will he have at the end of the year?

Ans. £164 12*s*. 6*d*.

36. What is the cost of 30 pieces of lead, each weighing 1*cwt*. 12*lb*. at the rate of 16*s*. 4*d*. the cwt.?

Ans. £27 2*s*. 6*d*.

37. The governor of a besieged place has provisions for 54 days at the rate of 2*lb*. of bread per ration, but is desirous to prolong the siege to 80 days in expectation of succour: in that case, what must b the ration of bread?

Ans. $1\frac{7}{20}$lb.

38. If a person pays half a guinea a week f *his board, how* long can he be boarded for £21?

Ans. 40 week

39. What is the value of a year's rent of 547 acres of land at the rate of 15*s.* 6*d.* the acre?

Ans. £423 18*s.* 6*d.*

40. If a person drink 20 bottles of wine per month, when it costs 2*s.* per bottle, how much can he drink without increasing the expense when it costs 2*s.* 6*d.* per bottle?

Ans. 16 bottles.

41. A merchant bought 21 pieces of cloth, each containing 40 yards, for which he paid $1260; he sold the cloth at $1,75 per yard: did he make or lose by the bargain?

Ans. he gained $210.

42. A cistern containing 200 gallons is filled by a pipe which discharges 3 gallons in 5 minutes: but the cistern has a leak which empties 1 gallon in 5 minutes. Now if the water begins to run in, when the cistern is empty, how long will it be in filling?

Ans. 8 hours 20 minutes.

43. What will be the cost of 895 feet of timber, at $6 per hundred feet?

$$\begin{array}{rl} 100 : & 895 :: 6 : \text{answer.} \\ & \underline{\quad\ \ 6} \\ 100) & \underline{5370} \\ & 53,70 \end{array}$$

Ans. $53,70.

In this example, 100 feet of timber, is to the given quantity 895 feet, as $6, the cost of 100 feet, to $53,70, the cost of 895 feet. If the timber had been sold at the rate of $6 per thousand feet, the cost of 895 feet would have been $5,37, for we have divided by 1000, instead of 100; th *should have* removed the separating point duct *three places to the left.* Hence, to fi of things sold by the 100, or 1000, we have the following

RULE.

Multiply the number by the price, and if the things be reckoned by the 100, *cut off two places from the right hand; if reckoned by the* 1000, *cut off three, and the figures to the left will be the answer in the same denomination as the given price.*

[illegible] What will be the cost of 1350 feet of boards at [illegible] per hundred?

Ans. $148,50.

[illegible] What will be the cost of 36578 bricks at [illegible] per thousand?

Ans. $237,75,+.

[illegible] What will be the cost of 6359 feet of boards at [illegible]?

Ans. $588,20,+.

[illegible] What will be the cost of [illegible] feet of timber at [illegible] per thousand?

Ans. $[illegible],+.

[illegible] What will [illegible] oranges cost at $5,50 per [illegible]?

Ans. $10[illegible],+.

OF QUESTIONS REQUIRING TWO STATEMENTS.

The answer to each of the above questions has been found by a single statement. Questions, however, frequently occur in which two or more statements are necessary. In most Arithmetics, such questions are arranged under a rule called Compound Proportion, or the Double Rule of Three. They [illegible] however, be answered by the rules already [illegible].

Ex. 1. If 16 men build 18 feet of wall in 12 days, how many men must be employed to build 72 feet in 8 days, working at the same rate?

The first question is, how long would it take the 16 men to build the 72 feet of wall?

It is evident that 18 feet of wall, is to 72 feet, as 12 days, the time necessary to build 18 feet, to 48 days, the time necessary to build 72 feet.

```
feet. feet.  days.   days.
 18 : 72 : : 12 : answer.
              72
             ----
              24
             84
           ------
          18)864(48 days.
              72
             ----
             144
             144
             ----
```

The second question is, if 16 men can build 72 feet of wall in 48 days, how many men are necessary to build it in 8 days?

Make 16 men the third term. Then as the same work is to be done in a less time, more men will be necessary; then the fourth term will be greater than the third, and hence 8 days are placed in the first term. § 109.

```
days. days.  men.    men.
  8 : 48 : : 16 : answer.
              48
             ----
             128
             64
            -----
            8)768(
          ---------
      Ans.    96 men.
```

2. If a family of 6 persons expend $300 in 8 months, how much will serve a family of 15 persons for 20 months?

First question. If $300 will support a family of 6 persons for 8 months, how many dollars will support *15 persons* for the *same time?*

```
persons. persons.    $      $
   6   :   15   : :  3[illegible]

                    6[illegible]

                  Ans.
```

Second question. If $750 will support a family of 15 persons for 8 months, how much will serve them for 20 months?

months.		*months.*		*$*		*$*
8	:	20	: :	750	:	*ans.*
				20		
				8)15000		
			Ans.	$1875		

3. If a man travel 217 miles in 7 days, travelling 6 hours a day, how far would he travel in 9 days, if he travelled 11 hours a day.

1st.	2d.
days. days. miles. miles.	*hours. hours. miles. miles.*
7 : 9 : : 217 : 279	6 : 11 : : 279 : $511\frac{3}{6}$
9	11
7)1953	6)3069
279	$511\frac{3}{6}$

Ans. $511\frac{3}{6}$ miles.

4. If the wages of 6 men for 14 days be $84, what will be the wages of 9 men for 11 days?

Ans. $99.

5. If 154 bushels of oats serve 14 horses for 44 days, how long would 406 bushels last 7 horses?

Ans. 232 days.

6. If 25 men can earn $6250 in 2 years, how long will it take 5 men to earn $11250?

Ans. 18 years.

7. If a barrel of beer last 7 persons 12 days, how much will be drank by 42 persons in a year?

Ans. 182*bar.* 18*gal.*

8. If 9 men can cut 36 acres of grass in 4 days, how many acres will 19 men cut in 11 days?

Ans. 209 acres.

9. If 25 persons consume 300 bushels of corn in
; how much will 139 persons consume in
t the same rate?

Ans. 11676bu.

10. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide in 4 days; in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet wide?

Ans. 36 days.

OF VULGAR FRACTIONS.

(Before proceeding farther let the pupil study carefully from § 56 to the end of § 59.)

§ 111. There are five kinds of Vulgar Fractions, Proper, Improper, Simple, Compound, and Mixed.

A proper fraction is one in which the numerator is less than the denominator; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. The value of every proper fraction is less than 1, § 59.

An improper fraction is one in which the numerator is equal to, or exceeds the denominator; as $\frac{2}{2}$, $\frac{4}{3}$, $\frac{7}{5}$, &c. Such fractions are called improper fractions because they are equal to or exceed unity. When the numerator is equal to the denominator the value of the fraction is 1: in every other case the value of an improper fraction is greater than 1.

A simple fraction is a single expression; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{2}$, $\frac{4}{3}$, &c. A simple fraction may be either proper or improper.

A compound fraction is a fraction of a fraction, or several fractions connected together with the word *of* between them; as $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{3}{4}$ of $\frac{2}{5}$ of 3, &c.

A mixed number is made up of a whole number and a fraction, as $3\frac{1}{2}$, $12\frac{1}{8}$, $16\frac{1}{7}$, &c. The whole numbers are sometimes called *integers*.

§ 112. The numerator and denominator of a fraction, *taken together*, are called the *terms* of the tion. *Hence, every fraction has two terms.*

§ 113. A whole number may be expressed fractionally, without altering its value, by writing 1 below it for a denominator: thus, 3 may be written $\frac{3}{1}$, 5 may be written $\frac{5}{1}$, 6 may be written $\frac{6}{1}$, &c.; when so written they are read 3 ones, 5 ones, 6 ones, &c.

§ 114. Since the denominator of a fraction shows into how many equal parts the unit has been divided, and the numerator how many of those equal parts are taken in the fraction § 57, the fraction $\frac{5}{8}$ will express that the unit has been divided into 8 equal parts and that 5 of those parts are taken. Each part of the unit will be expressed by $\frac{1}{8}$ and 5 of those parts by $\frac{1}{8}\times5$: hence $\frac{5}{8}=\frac{1}{8}\times5$. In the same way $\frac{7}{14}=\frac{1}{14}\times7$, that is $\frac{7}{14}$ is equal to $\frac{1}{14}$ repeated 7 times.

Again, $\frac{7}{14}\times3=\frac{1}{14}\times7\times3=\frac{1}{14}\times21=\frac{21}{14}$. Thus in multiplying the numerator of a fraction by any number we increase the number of parts expressed by the fraction as many times as there are units in the multiplier. We may therefore write

PROPOSITION I. *If the numerator of a fraction be multiplied by any number, the denominator remaining unchanged, the value of the fraction will be increased as many times as there are units in the multiplier.*

§ 115. If we take any fraction, as $\frac{12}{8}$, and divide the numerator by 2, it will change the fraction to $\frac{6}{8}$: if we divide it by 3, we shall have $\frac{4}{8}$, and if we divide it by 4, we shall have $\frac{3}{8}$. Now in each of the fractions $\frac{6}{8}$, $\frac{4}{8}$ and $\frac{3}{8}$ the *number* of equal parts expressed, is as *many times less than* 12, as there were units in the divisor; and as the same may be shown of all fractions, we write,

TION II. *If the numerator of a fraction be* *any number*, the denominator remaining *the value of* the fraction will be dimin- *ny times as there* are units in the divisor

§ 116. If we take any fraction, as $\frac{3}{5}$, and multiply the denominator by 4, it will change the fraction to $\frac{3}{20}$. Now in this last fraction the unit or single thing is divided into 20 equal parts, while in the first it is divided into 5 parts. We may consider that the 20 equal parts in the 2d case, have been obtained by dividing *each* of the parts of the first fraction into 4 equal parts: therefore, 1 part in the first fraction, is equal to 4 parts in the second. But the *number* of parts expressed in each fraction is the same; hence the second fraction is $\frac{1}{4}$ of the first: and as the same may be shown of any fraction, we write,

PROPOSITION III. *If the denominator of a fraction be multiplied by any number, the numerator remaining unchanged, the value of the fraction will be diminished as many times as there are units in the multiplier.*

§ 117. If we take any fraction as $\frac{4}{12}$, and divide the numerator by 6, it will change the fraction to $\frac{4}{2}$. Now in the second fraction the unit or single thing is divided into 2 parts, and if we divide each of these parts into 6 parts, we shall obtain the 12 parts into which the unit is divided in the first fraction: therefore, 1 part in the second fraction is equal to 6 parts in the first. But the number of parts expressed in each fraction is the same; hence, the second fraction is 6 times greater than the first, and as the same may be shown for any fraction, we write,

PROPOSITION IV. *If the denominator of a fraction be divided by any number, the numerator remaining unchanged, the value of the fraction will be increased as many times as there are units in the divisor.*

§ 118. It appears from Prop. I. that if the numerator of a fraction be multiplied by any number value of the fraction will be increased as many as there are units in the multiplier. It also appears from Prop. III. that if the denominator of a fraction be multiplied by any number, the value of the fraction will be *diminished* as many times as there are units in the multiplier. Therefore, when the numerator and denominator of a fraction are multiplied by the same number, the increase from multiplying the numerator, will be just equal to the decrease from multiplying the denominator; hence we have,

Proposition V. *If the numerator and denominator of a fraction be multiplied by the same number, the value of the fraction will remain unchanged.*

Thus, $\frac{3}{5}=\frac{3}{5}\times\frac{4}{4}=\frac{12}{20}$ and $\frac{5}{7}=\frac{5}{7}\times\frac{5}{5}=\frac{25}{35}$.

§ 119. It appears from Prop. II, that if the numerator of a fraction be divided by any number, the value of the fraction will be diminished as many times as there are units in the divisor. It also appears from Prop. IV, that if the denominator of a fraction be divided by any number, the value of the fraction will be increased as many times as there are units in the divisor. Therefore, when the numerator and denominator of a fraction are divided by the same number, the *decrease* from dividing the numerator will be just equal to the increase from dividing the denominator: hence we have,

Proposition VI. *If the numerator and denominator of a fraction be divided by the same number, the value of the fraction will remain unchanged.*

Thus, $\frac{8}{32}=\frac{2}{8}$ by dividing by 4: and $\frac{2}{8}=\frac{1}{4}$ by dividing by 2.

§ 120. *Any number greater than unity that will divide two or more numbers without a remainder is called their common divisor: and the greatest number that will so divide them, is called their greatest mmon divisor.*

Take, for example, the two numbers 142 and 994. The greatest common divisor cannot be greater than the least number 142. This number will divide itself:—let us see if it will also divide 994.

The number 142 exactly divides itself, giving a quotient of 1; it also divides 994 giving a quotient of 7. Therefore, 142 is the greatest common divisor.

142)142(1
      142

142)994(7
      994

The numbers 2 and 71 are *common* divisors of the two numbers since either of them will divide both of the numbers without a remainder. Two numbers may have several common divisors, but they have only one *greatest* common divisor.

Ex. 2. Take the two numbers 72 and 90.

Let us again see if the least number 72, is the greatest common divisor. After dividing we find a remainder of 18.

72)90(1
     72
great. c. div. 18)72(4
     72

Now if 18 will divide 72, it will also divide 90, for $90=72+18$, and 18 will be contained once more in $90=72+18$ than in 72: but 18 divides 72 without a remainder: therefore, 18 is the common divisor: hence we see *that the common divisor of two numbers must also be a common divisor between the least number and their remainder after division.* But 18 is the *greatest* common divisor; for, the greatest common divisor must be contained at least *once* more in 90 than in 72: hence, the greatest common divisor cannot be greater than the difference between the two numbers, which in this case is 18. Therefore, we have

§ 121. PROPOSITION VII. The greatest com
visor of two numbers is obtained by divid
greater by the less; then dividing the divis

remainder, and continuing to divide the last divisor by the last remainder until nothing remains. The last divisor will be the greatest common divisor sought.

Ex. 3. Find the greatest common divisor of the two numbers 63 and 81.

63)81(1
63
18)63(3
54
Gr. com. div. 9)18(2
18

PROOF.
9)63(7
63
9)81(9
81

4. Find the greatest common divisor of 315 and 405.

Ans. 45.

5. What is the greatest common divisor of the two numbers 2205 and 2835?

Ans. 315.

NOTE. § 122. If it be required to find the greatest common divisor of more than two numbers, find first the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on, for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

Ex. 6. What is the greatest common divisor of 246, 372 and 522?

Ans. 6.

7. What is the greatest common divisor of 492, 744 and 1044?

Ans. 12.

QUESTIONS.

111. How many kinds of Vulgar Fractions are there? *are they?* What is a proper fraction? Is its value greater or less than 1? What is an improper fraction? Why is it called improper? When is its value equal to 1? What is a simple fraction? What is a compound fraction? What is a mixed number?

§ 112. What are the numerator and denominator of a fraction, taken together, called? How many terms has every fraction?

§ 113. How may a whole number be expressed fractionally? Does this alter its value?

§ 114. What does the denominator of a fraction show? What does its numerator show? Repeat Prop. I.

§ 115. Repeat Prop. II.

§ 116. Repeat Prop. III

§ 117. Repeat Prop. IV.

§ 118. Repeat Prop. V.

§ 119. Repeat Prop. VI.

§ 120. What is the common divisor of two or more numbers? What is the greatest common divisor?

§ 121. How is the greatest common divisor found?

§ 122. How do you find the greatest common divisor of three or more numbers?

REDUCTION OF VULGAR FRACTIONS.

§ 123. Reduction of Vulgar Fractions is the method of changing the forms of the fractions without altering their values.

§ 124. A fraction is said to be in its lowest terms, when there is no number greater than 1 that will divide the numerator and denominator without a remainder.

CASE I.

§ 125. To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator, the quotient will be the whole number, and the remainder there be one, placed over the given denominator, form the fractional part.

Ex. 1. Reduce $\frac{84}{4}$ and $\frac{67}{9}$ to their equivalent whole or mixed numbers.

$$\begin{array}{ll} & 4)\underline{84} \\ \textit{Ans.} & 21 \end{array} \qquad\qquad \begin{array}{ll} & 9)\underline{67} \\ \textit{Ans.} & 7\frac{4}{9} \end{array}$$

It was shown in § 59, that the value of every fraction is equal to the quotient arising from dividing the numerator by the denominator: hence the value of the fraction is not changed by the reduction.

2. Reduce $\frac{99}{8}$ to a whole or mixed number.

Ans. $12\frac{3}{8}$.

3. In $\frac{15}{7}$ of yards of cloth, how many yards?

Ans. $2\frac{1}{7}$*yd.*

4. In $\frac{41}{8}$ of bushels, how many bushels?

Ans. $5\frac{1}{8}$*bu.*

5. If I give $\frac{1}{3}$ of an apple to each one of 15 children, how many apples do I give?

Ans. 5.

CASE II.

§ 126. To reduce a mixed number to its equivalent improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction; to the product add the numerator, and place the sum over the given denominator.

Ex. 1. Reduce $4\frac{4}{5}$ to its equivalent improper fraction. Here $4\times5=20$: then $20+4=24$; hence

$\frac{24}{5}$ is the equivalent fraction.

Ans. $\frac{24}{5}$.

This rule is the reverse of Case I. In the exam-
le $4\frac{4}{5}$ *we have the* integer number 4 and the frac-
1 whole thing is equal to 5 fifths, and
are equal to 20 fifths; to which, add

4 fifths and we have $\frac{24}{5}$, equal to the mixed number $4\frac{4}{5}$.

2. Reduce $47\frac{5}{6}$ to its equivalent improper fraction.
Ans. $\frac{287}{6}$.

3. Reduce $676\frac{37}{51}$, $874\frac{33}{9}$, $690\frac{47}{100}$, $367\frac{9}{104}$, to their equivalent improper fractions.
Ans. $\frac{34513}{51}$, $\frac{7899}{9}$, $\frac{69047}{100}$, $\frac{38177}{104}$.

CASE III

§ 127. To reduce a fraction to its lowest terms.

RULE.

Divide the numerator and denominator by any number that will divide them both without a remainder, and then divide the quotients arising in the same way until there is no number greater than 1 *that will divide them without a remainder.*

Or, find the greatest common divisor of the numerator and denominator and divide them by it. The value of the fraction will not be altered by reduction. § 119.

Ex. 1. Reduce $\frac{70}{175}$ to its lowest terms.

1st method.

$\frac{5)\,70}{5)\,175}=\frac{7)\,14}{7)\,35}=\frac{2}{5}$, which is the lowest term of the fraction, since no number greater than 1 will divide the numerator and denominator without a remainder.

2d method, by the common divisor.

$$\begin{array}{rl} 70)\,175\,(2 & \\ 140 & \\ \text{Greatest com. div. } 35)\,70\,(2 & \qquad \frac{35)\,70}{35)\,175}=\frac{2}{5} \quad A, \\ 70 & \end{array}$$

2. Reduce [illegible] to its lowest terms. Ans.

3. Reduce [illegible] to its lowest terms.

Ans. [illegible].

4. Reduce [illegible] to its lowest terms.

Ans. [illegible].

5. Reduce [illegible] to its lowest terms.

Ans. [illegible].

6. Reduce [illegible] to its lowest terms.

Ans. $\frac{117}{265}$.

7. Reduce $\frac{172}{1118}$ to its lowest terms.

Ans. $\frac{2}{13}$.

CASE IV.

§ 128. To reduce a whole number to an equivalent fraction having a given denominator.

RULE.

Multiply the whole number by the given denominator, and set the product over the said denominator.

Ex. 1. Reduce 6 to a fraction whose denominator shall be 4.

Here $6\times4=24$; therefore $\frac{24}{4}$ is the required fraction. It is plain that the fraction will in all cases be equál to the whole number, since it may be reduced to the whole number by Case I.

2. Reduce 15 to a fraction whose denominator shall be 9.

Ans. $\frac{135}{9}$.

3. Reduce 139 to a fraction whose denominator shall be 175.

Ans. $\frac{24325}{175}$.

CASE V.

§ *129.* To reduce a compound fraction to its equi-
t simple one.

us take the fraction $\frac{3}{4}$ of $\frac{5}{7}$.

fraction is equal to $3\times\frac{1}{4}$ of $\frac{5}{7}$. But $\frac{1}{4}$ of $\frac{5}{7}$

is equal to $\frac{5}{4\times7}=\frac{5}{28}$; but this fraction must be taken 3 times to form the first fraction; hence $\frac{3}{4}$ of $\frac{5}{7}=3\times\frac{1}{4}$ of $\frac{5}{7}=3\times\frac{5}{28}=\frac{15}{28}$, a result which is obtained by multiplying together the numerators and denominators of the compound fraction. When the compound fraction consists of more than two simple ones, two of them can be reduced to a simple fraction as above, and then this fraction may be reduced with the next, and so on. We therefore have the following

RULE.

I. *Reduce all mixed numbers to their equivalent improper fractions by Case II.*

II. *Then multiply all the numerators together for a numerator and all the denominators together for a denominator: their products will form the fraction sought.*

Ex. 2. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$ to a simple fraction.

Here $\frac{1}{2}\times\frac{1}{3}\times\frac{5}{7}=\frac{5}{42}$. *Ans.* $\frac{5}{42}$.

3. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here $\frac{5}{3}\times\frac{3}{6}\times\frac{6}{7}=\frac{90}{126}=\frac{10}{14}=\frac{5}{7}$ by dividing the numerator and denominator first by 9 and then by 2, as shown in Case III.

Or, $\frac{5}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{6}}\times\frac{\cancel{6}}{7}=\frac{5}{7}$, by cancelling the 3's and 6's in the numerator and denominator.

By cancelling or striking out the 3's we only divide the numerator and denominator of the fraction by 3; and in cancelling the 6's we divide by 6. *Hence the value of the fraction* is not affecte *striking out like figures*, which should alw *done* when they multiply the numerator a *minator.*

4. Reduce [illegible] of [illegible] of [illegible] to a simple fraction.

Here [illegible] *Ans.*

Or, [illegible] *Ans.*

5. Reduce $2\frac{1}{4}$ of $6\frac{1}{2}$ of 7 to a simple fraction.
Ans. $\frac{819}{8}=102\frac{3}{8}$.

6. Reduce 5 of $\frac{1}{3}$ of $\frac{1}{4}$ of 6 to a simple fraction.
Ans. $\frac{30}{12}=2\frac{1}{2}$.

CASE VI.

§ 130. To reduce fractions of different denominators to equivalent fractions having a common denominator.

RULE.

I. *Reduce compound fractions to simple ones, and whole or mixed numbers to improper fractions.*

II. *Then multiply each one of the numerators by all the denominators except its own, for the new numerators, and multiply all the denominators together for a common denominator: the common denominator placed under each of the new numerators will form the several fractions sought.*

Ex. 1. Reduce $\frac{1}{2}$, $\frac{7}{3}$, and $\frac{4}{5}$ to a common denominator.

$1\times3\times5=15$ the new numerator for $\frac{1}{2}$.
$7\times2\times5=70$ do do $\frac{7}{3}$.
$4\times3\times2=24$ do do $\frac{4}{5}$.
and $2\times3\times5=30$, the common denominator.

Therefore, $\frac{15}{30}$, $\frac{70}{30}$ and $\frac{24}{30}$, are the equivalent fractions.

It is plain, that this reduction does not alter the
of the several fractions, since the numerator
minator of each are multiplied by the same
§ 118.

When the numbers are small the whole work may be performed mentally.

Thus, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{5} = \frac{20}{40}$, $\frac{10}{40}$, $\frac{16}{40}$.

Here we find the first numerator by multiplying 1 by 4 and 5; the second, by multiplying 1 by 2 and 5; the third by multiplying 2 by 4 and 2; and the common denominator by multiplying 2, 4 and 5 together.

Ex. 3. Reduce $2\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{1}{7}$ to a common denominator.

$2\frac{1}{3}=\frac{7}{3}$; and $\frac{1}{2}$ of $\frac{1}{7}=\frac{1}{14}$.

$\frac{7}{3}$, $\frac{1}{14}=\frac{98}{42}$, and $\frac{3}{42}$; the answers.

4. Reduce $5\frac{1}{2}$, $\frac{6}{7}$ of $\frac{1}{3}$, and 4, to a common denominator.

Ans. $\frac{77}{14}$, $\frac{4}{14}$, and $\frac{56}{14}$.

5. Reduce $\frac{7}{8}$, $\frac{135}{75}$, and 37, to a common denominator.

Ans. $\frac{525}{600}$, $\frac{1080}{600}$, and $\frac{22200}{600}$.

6. Reduce 4, $\frac{31}{25}$, $\frac{62}{2}$, to a common denominator.

Ans. $\frac{200}{50}$, $\frac{62}{50}$, and $\frac{1550}{50}$.

NOTE. § 131. It is often convenient to reduce fractions to a common denominator by multiplying the numerator and denominator of each fraction by such a number as shall make the denominators the same in both.

7. Let it be required to reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

We see at once that if we multiply the numerator and denominator of the first fraction by 3, and the numerator and denominator of the second

that they will have a common denominator.

The two fractions will be reduced to $\frac{3}{6}$

8. Reduce $\frac{1}{5}$ and $\frac{1}{3}$ to a common denominator.

Here, $\frac{1}{5}\times\frac{3}{3}=\frac{3}{15}$; and $\frac{1}{3}\times\frac{5}{5}=\frac{5}{15}$.

Ans. $\frac{3}{15}$ and $\frac{5}{15}$.

QUESTIONS.

§ 123. What is Reduction of Vulgar Fractions?

§ 124. When is a fraction said to be in its lowest terms?

§ 125. How do you reduce an improper fraction to its equivalent whole or mixed number?

§ 126. How do you reduce a mixed number to its equivalent fraction? Will the fraction be proper, or improper?

§ 127. How do you reduce a fraction to its lowest terms? Will the value be altered by the reduction? Why?

§ 128. How do you reduce a whole number to a fraction having a given denominator?

§ 129. How do you reduce a compound fraction to a simple one? When you have the same multiplier in the numerator and denominator, what do you do? Does this alter the value of the fraction?

§ 130. How do you reduce fractions to a common denominator? Does this reduction change the values of the several fractions? Why not?

§ 131. What is a second method of reducing fractions to a common denominator?

REDUCTION OF DENOMINATE FRACTIONS.

§ 132. We have seen § 71, that a denominate number is one in which the kind of unit is denominated, or expressed. For the same reason, a denominate fraction is one which expresses the *kind of unit* that has been divided. Such unit is called the unit of the fraction. Thus $\frac{2}{3}$ of a £ is a denominate fraction. It expresses that one £ is the unit which has been divided, that this unit has been divided into 3 equal parts, and that 2 of these parts are taken in the fraction.

The fraction $\frac{3}{8}$ of a shilling is also a denominate fraction, in which the unit that has been divided is one shilling. These two fractions are of different denominations, the unit of the first being one pound, and that of the second, one shilling.

Fractions are therefore of different denominations when they express parts of different units, and of the same denomination when they express parts of the same unit.

§ 133. *Reduction of denominate fractions consists in changing their denominations without altering their values.* For example, to reduce $\frac{2}{3}$ of a £. to the denomination of shillings, or $\frac{3}{8}$ of a shilling to the denomination of pounds.

CASE I.

§ 134. To reduce a denominate fraction from a lower to a higher denomination.

RULE.

I. *Consider how many units of the given denomination make one unit of the next higher, and place* 1 *over that number forming a second fraction.*

II. *Then consider how many units of the second denomination make one unit of the denomination next higher, and place* 1 *over that number forming a third fraction; and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction; then reduce the compound fraction to a simple one by Case V.*

Ex. 1. Reduce $\frac{1}{3}$ of a penny to the fraction of a £.

Here $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$=£$\frac{1}{720}$. The given fraction is $\frac{1}{3}$ of a penny. But one penny is equal to $\frac{1}{12}$ of a shilling: hence $\frac{1}{3}$ of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of a shilling. But one shilling is equal to $\frac{1}{20}$

£.: hence $\frac{1}{3}$ of a penny is equal to $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £$=\frac{1}{720}$£. The reason of the rule is therefore evident.

2. Reduce $\frac{3}{7}$ of an inch to the denomination of yards.

Here $\frac{3}{7}$ of $\frac{1}{12}$ of $\frac{1}{3}=\frac{3}{252}$ yard, the answer.

3. Reduce $\frac{3}{4}$*oz.* avoirdupois to the denomination of tons.

Ans. $\frac{3}{143360}$ *T.*

4. Reduce $\frac{5}{8}$ of a pint to the fraction of a hogshead.

Ans. $\frac{5}{4536}$*hhd.*

CASE II.

§ 135. To reduce a denominate fraction from a higher to a lower denomination.

RULE.

I. *Consider how many units of the next lower denomination make one unit of the given denomination, and place 1 under that number forming a second fraction.*

II. *Then consider how many units of the denomination still lower make one unit of the second denomination and place 1 under that number forming a third fraction, and so on, to the denomination to which you would reduce.*

III. *Connect all the fractions together, forming a compound fraction. Then reduce the compound fraction to a simple one by Case V.*

Ex. 1. Reduce $\frac{1}{7}$£ to the denomination of pence.

Here $\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}$*d.* *Ans.*

In this example $\frac{1}{7}$ of a £. is equal to $\frac{1}{7}$ of 20 shillings. But 1 shilling is equal to 12 pence; hence f a £$=\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}$*d.* Hence the reason he rule is manifest.

2. Reduce $\frac{4}{7}$*cwt.* to the fraction of a pound.
Ans. $\frac{448}{7}$*lb.*

3. Reduce $\frac{2}{15}$ of a £. to the fraction of a penny.
Ans. $\frac{32}{1}$*d.*

4. Reduce $\frac{1}{3}$ of a day to the fraction of a minute.
Ans. 480*m.*

5. Reduce $\frac{3}{8}$ of an acre to the fraction of a pole.
Ans. $\frac{480}{8}$P.

CASE III.

§ 136. To find the value of a fraction in integers of a less denomination.

RULE.

Multiply the numerator by that number which makes one of the next lower denomination, and divide the product by the denominator. If there be a remainder, multiply it by that number which makes one of the denomination still less, and divide again by the denominator. Proceed in the same way to the lowest denomination. The several quotients being connected together, will form the equivalent denominate number.

Ex. 1. What is the value of $\frac{2}{3}$ of a £.?

```
    2
    20
  ------
  3)40
  -------
   13s . . . 1 Remainder
               12
             -----
             3)12
             -----
               4d
```

Ans. 13s 4d.

2. What is the value of $\frac{3}{4}$lb. troy?
Ans. 9oz. 12

3. What is the value of $\frac{5}{16}$ of a *cwt.*?
Ans. 1*qr.* 7*lb.*

4. What is the value of $\frac{5}{8}$ of an acre?
Ans. 2R. 20P.

CASE IV.

§ 137. To reduce a denominate number to a fraction of a given denomination.

RULE.

Reduce the number to the lowest denomination mentioned in it: then if the reduction is to be made to a denomination still less, reduce as in Case II.; but if to a higher denomination reduce as in Case I.

Ex. 1. Reduce 4*s* 7*d* to the fraction of a £.

4
12 — 55 of $\frac{1}{20}$ of $\frac{1}{12}=\frac{55}{240}$ of a £. *Ans.*
55*d* by adding in the 7*d*.

2. Reduce 2 feet 2 inches to the fraction of a yard.
Ans. $\frac{13}{18}$*yd.*

3. Reduce 3 gallons 2 quarts to the fraction of a hogshead.
Ans. $\frac{7}{126}$*hhd.*

4. Reduce 1*qr.* 7*lb.* to the fraction of a hundred.
Ans $\frac{5}{16}$*cwt.*

QUESTIONS.

§ 132. What is a denominate number? What is a denominate fraction? What is the unit of a fraction? When are fractions of different denominations? When of the same *denomination*?

What is reduction of denominate fractions?
w do you reduce a denominate fraction from a
gher denomination?

§ 135. How do you reduce a denominate fraction from a higher to a lower denomination?

§ 136. How do you find the value of a fraction in integers of a less denomination?

§ 137. How do you reduce a denominate number to a fraction of a given denomination?

ADDITION OF VULGAR FRACTIONS.

§ 138. Addition of integer numbers teaches how to express all the units of several numbers by a single number.

Addition of fractions teaches how to express the values of several fractions by a single fraction.

It is plain, that we cannot add fractions so long as they have different units: for, $\frac{1}{2}$ of a £. and $\frac{1}{2}$ of a shilling make neither 1£ nor 1 shilling.

Neither can we add parts of the same unit unless they be like parts; for $\frac{1}{3}$ of a £. and $\frac{1}{4}$ of a £. make neither $\frac{2}{3}$ of a £. nor $\frac{2}{4}$ of a £. But $\frac{1}{3}$ of a £. and $\frac{1}{3}$ of a £. may be added: they make $\frac{2}{3}$ of a £. So, $\frac{1}{4}$ of a £. and $\frac{1}{4}$ of a £. make $\frac{2}{4}$ of a £.

Hence before fractions can be added, two things are necessary.

1st. *That the fractions be reduced to the same denomination.*

2d. *That they be reduced to a common denominator.*

CASE I.

§ 139. When the fractions to be added are of the same denomination and have a common denominator.

RULE.

Add the numerators together and place their sum over the common denominator: then reduce the fraction to its lowest terms, or to its equivalent mixed number.

Ex. 1. Add [illegible] together.

Here 1+3+6+3=13, the sum of the numerators; Hence $\frac{13}{2}$ is the sum of the fractions.

Ans. $\frac{13}{2}=6\frac{1}{2}$.

2. Add [illegible] of a £., [illegible] of a £. and [illegible] of a £. together.

Ans. [illegible] of a £.=2[illegible]£.

CASE II.

§ 140. When the fractions are of the same denomination but have different denominators.

RULE.

Reduce compound fractions to simple ones, mixed numbers to improper fractions, and all the fractions to a common denominator. Then add them as in Case I.

Ex. 1. Add $\frac{6}{2}$, $\frac{4}{3}$, $\frac{2}{5}$ together.

$$\left.\begin{array}{l}6\times3\times5=90\\4\times2\times5=40\\2\times3\times2=12\end{array}\right\}\text{Numerators.}$$

$2\times3\times5=30$ common denominator.

Hence, $\frac{6}{2}+\frac{4}{3}+\frac{2}{5}=\frac{90}{30}+\frac{40}{30}+\frac{12}{30}=\frac{90+40+12}{30}=\frac{142}{30}$ $=4\frac{22}{30}=4\frac{11}{15}$.

Ans. $4\frac{11}{15}$.

2. Add [illegible] of a £., [illegible] of a £. and [illegible] of a £ together.

Ans. $\pounds\frac{453}{378}=\pounds1\frac{75}{378}=\pounds1\frac{25}{126}$.

NOTE. § 141. When there are mixed numbers, instead of reducing them to improper fractions we may add the whole numbers and the fractional parts separately, and then add their sums.

Ex. 3. Add 19[illegible], 6[illegible] and 4[illegible] together.

19+6+4=29 the sum of the whole numbers [illegible] the sum of [illegible]onal parts. Hence, 29+1[illegible]=30[illegible].

Ans. 30[illegible].

CASE III.

§ 142. When the fractions are of different denominations.

RULE.

Reduce the fractions to the same denomination. Then reduce all the fractions to a common denominator, and then add them as in Case 1.

Ex. 1. Add $\frac{2}{3}$ of a £. to $\frac{5}{6}$ of a shilling.

$\frac{2}{3}$ of a £.$=\frac{2}{3}$ of $\frac{20}{1}=\frac{40}{3}$ of a shilling:

Then, $\frac{40}{3}+\frac{5}{6}=\frac{240}{18}+\frac{15}{18}=\frac{255}{18}s=\frac{85}{6}s=14s\ 2d.$

Or, the $\frac{5}{6}$ of a shilling might have been reduced to the fraction of a £. thus,

$\frac{5}{6}$ of $\frac{1}{20}=\frac{5}{120}$ of a £.$=\frac{1}{24}$ of a £.

Then, $\frac{2}{3}+\frac{1}{24}=\frac{48}{72}+\frac{3}{72}=\frac{51}{72}$ of a £.: which being reduced by § 136, gives 14*s* 2*d*.

Ans. 14*s* 2*d*.

2. Add $\frac{3}{8}$ of a yard to $\frac{5}{9}$ of an inch.

Ans. $\frac{253}{648}$yds. or $14\frac{1}{18}$inches.

3. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together.

Ans. 2*da.* $14\frac{1}{2}$*hr.*

4. Add $\frac{4}{7}$ of a cwt., $8\frac{5}{6}$*lb.* and $3\frac{9}{16}$*oz.* together.

Ans. 2*qr.* 17*lb.* $1\frac{7}{30}$*oz.*

5. Add $1\frac{1}{4}$ miles, $\frac{7}{16}$ furlongs, and 30 rods together.

Ans. 1*m.* 3*fur.* 18*rd.*

NOTE. § 143. The value of each of the fractions may be found separately, and their several values then added.

Ex. 6. Add $\frac{2}{3}$ of a year, $\frac{1}{3}$ of a week, and day together.

$\frac{3}{5}$ of a year $=\frac{3}{5}$ of $\frac{365}{1}$ days$=219$ days
$\frac{1}{3}$ of a week$=\frac{1}{3}$ of 7 days $=$ 2 days, 8 hours
$\frac{1}{8}$ of a day $=$ 3 hours.

Ans. 221*da.* 11*hr.*

7. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together. *Ans.* 1540*yd.* 2*ft.* 9*in.*

8. Add $\frac{3}{4}$ of a cwt., $\frac{43}{2}$*lb.* 13*oz.* and $\frac{1}{2}$ of a cwt. 6*lb.* together.

Ans. 1*cwt.* 1*qr.* 27*lb.* 13*oz.*

QUESTIONS.

§ 138. What is addition of integer numbers? What is addition of fractions? Can fractions be added while they have different units? If they have the same unit can they be added if the parts are unlike? What then is necessary before fractions can be added?

§ 139. How do you add when the fractions have a common denominator?

§ 140. How do you add when the fractions have different denominators?

§ 141. When there are mixed numbers, how may the addition be performed?

§ 142. How do you add when the fractions are of different denominations?

§ 143. In what other way may they be added?

SUBTRACTION OF VULGAR FRACTIONS.

§ 144. Subtraction of Vulgar Fractions teaches how to take a less fraction from a greater.

CASE I.

§ 145. When the fractions are of the same denomination, and have a common denominator.

RULE.

t the less numerator from the greater and
fference over the common denominator.

Ex. 1. What is the difference between $\frac{3}{8}$ and $\frac{2}{8}$? 3—2=1: hence $\frac{1}{8}$ is the answer.

2. What is the difference between $\frac{125}{365}$ and $\frac{67}{365}$? *Ans.* $\frac{58}{365}$.

CASE II.

§ 146. When the fractions are of the same denomination, but have different denominators.

RULE.

Reduce mixed numbers to improper fractions, compound fractions to simple ones, and all the fractions to a common denominator: then subtract them as in Case I.

Ex. 1. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Here, $\frac{5}{6}-\frac{1}{3}=\frac{5}{6}-\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$ answer.

2. What is the difference between $12\frac{1}{2}$ of $\frac{1}{6}$ and 2? *Ans.* $\frac{1}{12}$.

3. What is the difference between $2\frac{1}{2}$ of a £., and $\frac{3}{15}$ of a £.?

Ans. £2 6*s*.

CASE III.

§ 147. When the fractions are of different denominations.

RULE

Reduce the fractions to the same denomination: then reduce them to a common denominator, after which subtract as in Case I.

Ex. 1. What is the difference between $\frac{1}{2}$ of a £., and $\frac{1}{3}$ of a shilling?

$\frac{1}{3}$ of a shilling $=\frac{1}{3}$ of $\frac{1}{20}=\frac{1}{60}$ of a £.

Then, $\frac{1}{2}-\frac{1}{60}=\frac{30}{60}-\frac{1}{60}=\frac{29}{60}$ of a £=9*s* 8*d*.

2. What is the difference between $\frac{1}{2}$ of a day $\frac{2}{3}$ of a second?

Ans. 11*hr*. 59*m*. 59

3. What is the difference between $\frac{2}{3}$ of a rod and $\frac{3}{7}$ of an inch?

Ans. 10*ft.* 11$\frac{4}{7}$*in.*

4. From 1$\frac{3}{4}$ of a lb. troy weight, take $\frac{1}{6}$ of an ounce.

Ans. 1*lb.* 8*oz.* 16*pwt.* 16*gr.*

6. What is the difference between $\frac{4}{15}$ of a hogshead, and $\frac{4}{13}$ of a quart?

Ans. 16*gal.* 2*qt.* 1*pt.* 3$\frac{9}{65}$*gi.*

QUESTIONS.

§ 144. What does Subtraction of Vulgar Fractions teach?

§ 145. How do you subtract when the fractions are of the same denomination, and have a common denominator?

§ 146. How do you subtract when the fractions have different denominators?

§ 147. What do you do when the fractions are of different denominations?

MULTIPLICATION OF VULGAR FRACTIONS.

§ 148. When we multiply by a whole number we repeat the multiplicand as many times as there are units in the multiplier, § 28.

Therefore, if it be required to multiply a fraction, as $\frac{1}{8}$, by 4, it is necessary to increase the fraction as many times as there are units in 4; which, as shown in § 114 and § 117, may be done either by multiplying the numerator, or dividing the denominator by 4.

CASE I.

§ 149. To multiply a fraction by a whole number.

RULE.

Multiply the numerator, or divide the denominator by the whole number.

Multiply $\frac{4}{12}$ by 6.

$\frac{24}{12}=2$. Or $\frac{4}{12}\times6=\frac{4}{2}=\frac{2}{1}=2$ Ans.

2. Multiply $\frac{37}{144}$ by 12.

Ans. $3\frac{1}{12}$.

3. Multiply $\frac{47}{49}$ by 7.

Ans. $6\frac{5}{7}$.

§ 150. When we multiply by a fraction it is required to repeat the multiplicand as many times as there are *units* in the fraction.

For example, to multiply 8 by $\frac{3}{4}$ is to repeat 8, $\frac{3}{4}$ times; that is, to take $\frac{3}{4}$ of 8, which is 2.

Hence, *when the multiplier is less than* 1 *we do not take the whole of the multiplicand, but only such a part of it as the fraction is of unity.* For example, if the multiplier be one half of unity, the product will be half the multiplicand: if the multiplier be $\frac{1}{3}$ of unity, the product will be one third of the multiplicand, &c.

Ex. 1. Let it be required to multiply $\frac{3}{4}$ by $\frac{5}{7}$. Here $\frac{3}{4}$ is to be taken $\frac{5}{7}$ times. But $\frac{5}{7}=5\times\frac{1}{7}$; hence $\frac{3}{4}$ is to be multiplied by 5, and then $\frac{1}{7}$ of the product taken. But $\frac{3}{4}\times5=\frac{15}{4}$, and $\frac{15}{4}\div7=\frac{15}{4\times7}=\frac{15}{28}$ § 116. Hence, for multiplying one fraction by another we have the following

RULE.

§ 151. *Reduce all the mixed numbers to improper fractions, and all compound fractions to simple ones: then multiply the numerators together for a numerator, and the denominators together for a denominator.*

Ex. 1. Multiply $\frac{1}{6}$ of $\frac{3}{7}$ by $8\frac{1}{3}$.

$\frac{1}{6}$ of $\frac{3}{7}=\frac{3}{42}$; and $8\frac{1}{3}=\frac{25}{3}$.

Hence, $\frac{3}{42}\times\frac{25}{3}=\frac{25}{112}$.

Ans. $\frac{25}{42}$.

2. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$.

Ans. $\frac{21}{24}=7$

3. Multiply $\frac{15}{13}$ by $\frac{7}{9}$ of 9.

Ans. $8\frac{1}{13}$.

4. Multiply $\frac{1}{3}$ of 3 of $\frac{1}{6}$ by $15\frac{1}{6}$.

Ans. $2\frac{19}{36}$.

5. Multiply $\frac{5}{6}$ by $\frac{2}{3}$ of $\frac{6}{7}$.

Ans. $\frac{10}{21}$.

6. Required the product of 6 by $\frac{2}{3}$ of 5.

Ans. 20.

7. Required the product of $\frac{2}{3}$ of $\frac{3}{5}$ by $\frac{5}{6}$ of $3\frac{2}{7}$.

Ans. $\frac{23}{21}$.

8. Required the product of $3\frac{2}{7}$ by $4\frac{14}{33}$.

Ans. $14\frac{124}{231}$.

9. Required the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$ and $4\frac{1}{6}$.

Ans. $2\frac{8}{21}$.

NOTE. § 152. In multiplying by a mixed number, we may first multiply by the integer, then multiply by the fraction, and then add the two products together. This is the best method when the numerator of the fraction is 1.

Ex. 1. Multiply 26 by $3\frac{1}{2}$.

We first multiply 26 by 3: the product is 78. Afterwards we multiply 26 by $\frac{1}{2}$; the product is 13: hence the entire product is 91.

$$\begin{array}{r} 26 \\ 3 \\ \hline 78 \\ 26\times\frac{1}{2}=13 \\ \hline 91 \end{array}$$ *Ans.*

Ex. 2. Multiply 48 by $8\frac{1}{6}$.

$$\begin{array}{r} 48\times8=384 \\ 48\times\frac{1}{6}=\ \ 8 \\ \hline \textit{Ans.}\ \ 392 \end{array}$$

3. Multiply 67 by $9\frac{1}{12}$.

Ans. $608\frac{7}{12}$.

4. Multiply 842 by $7\frac{1}{9}$.

Ans. $5987\frac{1}{9}$

QUESTIONS.

§ 148. If we multiply by a whole number, how many times do we repeat the multiplicand? If we multiply a fraction by a whole number, how many times do we increase the fraction?

§ 149. How do you multiply a fraction by a whole number?

§ 150. When you multiply by a fraction what is required? When the multiplier is less than 1, how much of the multiplicand do you take?

§ 151. How do you multiply one fraction by another?

§ 152. How may you multiply by a mixed number? When is this the best method?

APPLICATIONS.

1. What will 7 yards of cloth cost at $\$\frac{3}{4}$ per yard?

Ans. $\$5\frac{1}{4}$.

2. What will 32 gallons of brandy cost, at $\$1\frac{1}{8}$ per gallon?

Ans. $36.

3. If 1*lb.* of tea cost $\$1\frac{1}{4}$, what will $6\frac{1}{4}$*lb.* cost?

Ans. $\$7\frac{13}{16}$.

4. What will be the cost of $17\frac{1}{2}$ yards of cambric at $2\frac{1}{2}$ shillings per yard?

Ans. £2 3*s* 9*d*.

5. What will $15\frac{1}{16}$ barrels of cider come to at $3 per barrel?

Ans. $\$45\frac{3}{16}$

DIVISION OF VULGAR FRACTIONS.

§ 153. We have seen § 41, that division of integer numbers explains the manner of finding how many times a less number is contained in a greater.

In division of fractions, the divisor may be larger than the dividend, **in which case the quotient will *be less* than 1.**

For example, divide 1 apple into 4 equal parts. Here it is plain, that each part will be $\frac{1}{4}$; or that the dividend will contain the divisor but $\frac{1}{4}$ times.

Again, divide $\frac{1}{2}$ of a pear into 6 equal parts. It is plain, that 1 will contain 6, $\frac{1}{6}$ times; therefore $\frac{1}{2}$ will contain 6 but $\frac{1}{12}$ times: hence $\frac{1}{12}$ would be one of the equal parts of the pear.

§ 154. When we divide a fraction by a whole number, we are to divide the fraction into as many equal parts as there are units in the divisor, and this may be done, as was shown in § 115, and § 117, either by dividing the numerator, or multiplying the denominator by the whole number.

CASE I.

§ 155. To divide a fraction by a whole number.

RULE.

Divide the numerator or multiply the denominator by the whole number.

Ex. 1. Divide [illegible] by 2.

Here [illegible] ÷ 2 = [illegible] : or [illegible] ÷ 2 = [illegible] = [illegible] = [illegible] Ans.

2. Divide [illegible] by 9.

Ans. [illegible] = [illegible].

3. Divide [illegible] by 15.

Ans. [illegible] = [illegible].

CASE II.

§ 156. To divide one fraction by another.

Ex. 1. Divide [illegible] by [illegible].

The [illegible] [illegible] the divisor were 3, the quotient would be [illegible] (§ 117). But since the divisor is only [illegible] the true quotient will be [illegible] [illegible] or it is evident that the fraction is [illegible] more times than 1 would [illegible]

But since the value of a fraction is equally affected by multiplying the denominator or dividing the numerator, it follows, that we might have divided the numerator 10 by 5, and the denominator 24 by 8, and obtained the same result $\frac{2}{3}$. Hence we have the following

RULE.

Reduce compound fractions to simple ones, and whole numbers to improper fractions; then divide the numerator by the numerator, and the denominator by the denominator, if they will exactly divide; but if not, invert the terms of the divisor, and then multiply the divisor and dividend together.

Ex. 1. Divide $\frac{16}{90}$ by $\frac{8}{5}$.

Divisor $\frac{8}{5})\frac{16}{90}=\frac{2}{18}=\frac{1}{9}$ *Ans.*

Or $\frac{16}{90}\div\frac{8}{5}=\frac{16}{90}\times\frac{5}{8}=\frac{80}{720}=\frac{1}{9}$ *Ans.*

2. Divide $\frac{1}{2}$ by $\frac{1}{16}$.

$\frac{1}{2}\div\frac{1}{16}=\frac{1}{2}\times\frac{16}{1}=\frac{16}{2}=8$ *Ans.*

§ 157. When the dividend is just equal to the divisor the quotient is 1. But when the dividend is greater than the divisor, the quotient will be greater than 1; and when the dividend is less than the divisor, the quotient will be less than 1. When the dividend is greater than the divisor, the quotient will be just as many times greater than unity as the dividend is greater than the divisor; and when the dividend is less than the divisor, the quotient will be just as many times less than unity as the dividend is less than the divisor. In the example above, the dividend $\frac{1}{2}$ is 8 times greater than $\frac{1}{16}$: *the quotient* is right, being 8 times greater *unity.*

QUESTIONS.

§ 153. does division of whole numbers explain? In **fractions,** he divisor be larger than the dividend? In **such case** will 1e quotient be greater or less than 1?

§ 154. Wha. is required when a fraction is to be divided by **a whole** number? How may the division be made?

§ 155. How o you divide a fraction by a whole number?

§ 156. How . you divide one fraction by another?

§ 157. When the divisor and dividend are equal, what is **the value** of tl quotient? When the dividend exceeds the **divisor,** will tl r less than 1? How **many** times? less than the divisor **will the** quotie many times?

3. Divide $\frac{1}{8}$ by $\frac{1}{7}$.

Ans. $\frac{7}{8}$.

4. Divide $\frac{1}{5}$.

Ans. $29\frac{1}{4}$.

5. Divide $16\frac{1}{2}$ of

Ans. $1\frac{19}{58}$.

6. Divide $44\frac{1}{33}$ by $\frac{213}{333}$.

Ans. $68\frac{5877}{7029}$.

7. Divide $371\frac{1}{2}$ by $1\frac{1}{414}$.

Ans. $370\frac{251}{415}$.

8. Divide $\frac{64}{111}$ by $\frac{23}{1213}$.

Ans. $30\frac{1042}{2553}$.

APPLICATION

1. If 7*lb.* of sugar cost $\frac{44}{75}$ of a dollar, what is the price per pound?

$\frac{44}{75}\div 7=\frac{44}{525}$ of a dollar; or $\frac{44}{525}$ of $\frac{100}{1}$ cents $=\frac{4400}{525}=8\frac{8}{21}$ cents.

Ans. $8\frac{8}{21}$ *cents.*

? of a dollar will pay for $10\frac{1}{4}$*lb.* of nails, is the price per pound?

Ans. $\frac{2}{45}$dolls.$=4\frac{4}{9}$cts.

3. If $\frac{4}{7}$ of a yard of cloth cost \$3, what is the price per yard?

Ans. \$$5\frac{1}{4}$.

4. If \$$21\frac{1}{2}$ will buy $7\frac{13}{16}$ barrels of apples, how much are they per barrel?

Ans. \$$2\frac{94}{125}$.

5. If $4\frac{1}{7}$ gallons of molasses cost \$$2\frac{5}{8}$, how much is it per quart?

Ans. $15\frac{195}{232}$*cts.*

6. If $1\frac{1}{8}$*hhd.* of wine cost \$$250\frac{1}{3}$, how much is the wine per quart?

Ans. \$$\frac{1502}{1701}=88\frac{512}{1701}$*cts.*

DECIMAL FRACTIONS.

§ 158. If the unit 1 be divided into 10 equal parts the parts are called *tenths*, because each part is one tenth of unity.

If the unit 1 be divided into one hundred equal parts, the parts are called *hundredths*, because each part is one hundredth of unity.

If the unit 1 be divided into one thousand equal parts, the parts are called *thousandths*, because each part is one thousandth of unity: and we have similar expressions for the parts, when the unit is divided into ten thousand, one hundred thousand, &c. equal parts.

The division of the unit into tenths, hundredths, thousandths, &c. forms a system of numbers called *Decimal Fractions*.

DECIMAL FRACTIONS.

Four tenths, - - - -	$\frac{4}{10}$.
Six tenths, - - - -	$\frac{6}{10}$.
Forty-five hundredths, - -	$\frac{45}{100}$.
125 thousandths, - - -	$\frac{125}{1000}$.
1047 ten thousandths, - -	$\frac{1047}{10000}$

From which we see, that in each case the denominator gives denomination to the fraction; that is, determines whether the parts are tenths, hundredths, thousandths, &c.

§ 159. The denominators of decimal fractions are seldom set down. The fractions are usually expressed by means of a point, or comma, placed at the left of the numerator.

Thus, four tenths, - - -	,4
forty-five hundredths, - -	,45
100 and twenty-five hundredths,	,125
1047 ten thousandths, - -	,1047

The denominator of every decimal fraction, however, is always understood. *It is a unit* 1 *with as many ciphers annexed as there are places of figures in the numerator.*

The place next to the decimal point is called tenths; the next place to the right, the place of hundredths; the next, the place of thousandths; and so on for places further to the right according to the following Table.

DECIMAL NUMERATION TABLE.

Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousan.	Millionths.	Tens of millionths.	Read
,4							4 tenths.
,6	4						64 hundredths.
,0	6	4					64 thousandths.
.6	7	5	4				6754 ten thousandths.
		2	3	4			1234 hundred thousandth
		7	6	5	4		7654 millionths.
		1	3	6	0	4	43604 ten millionths.

§ 160. Decimal fractions are numerated from the left hand to the right, beginning with the tenths, hundredths, &c. as in the table.

The 64 hundredths is made up of 6 tenths and 4 hundredths; and the 64 thousandths is made up of 6 hundredths and 4 thousandths. Had it been required to express 4 tenths, 4 hundredths, &c. we should have written

Four tenths, - - -	,4.
Four hundredths, - -	,0 4.
Four thousandths, - -	,0 0 4.
Four ten thousandths, - -	,0 0 0 4.
Four hundred thousandths, -	,0 0 0 0 4.
Four millionths, - - -	,0 0 0 0 0 4.
Four ten millionths, - -	,0 0 0 0 0 0 4.

Here we see, that the same figure expresses different values, according to the place which it occupies.

But $\frac{1}{10}$ of $\frac{4}{10}$ is equal to $\frac{4}{100}$ $=,04.$
- $\frac{1}{10}$ of $\frac{4}{100}$ - $\frac{4}{1000}$ $=,004.$
- $\frac{1}{10}$ of $\frac{4}{1000}$ - $\frac{4}{10000}$ $=,0004.$
- $\frac{1}{10}$ of $\frac{4}{10000}$ - $\frac{4}{100000}$ $=,00004.$
- $\frac{1}{10}$ of $\frac{4}{100000}$ - $\frac{4}{1000000}$ $=,000004.$
- $\frac{1}{10}$ of $\frac{4}{1000000}$ - $\frac{4}{10000000}$ $=,0000004.$

§ 161. *Therefore the value of the place diminishes in a tenfold proportion from the left hand to the right.*

Hence, ten of the parts in any one of the places, are equal to one of the parts in the place next to the left; that is, ten thousandths make one hundredth, ten hundredths make one tenth, and ten tenths a unit 1.

This law of increase from the right hand towards *the left*, is the same as in whole numbers. T
fore, whole numbers and decimal fractions
united by placing the decimal point between

Thus,	36, 95 -	thirty-six, and 95 hundredths.
	127, 4 -	127 and four tenths.
	163, 03 -	163 and three hundredths.
	627, 0047	627 and 47 ten thousandths.

The whole numbers increase in a tenfold proportion from the decimal point towards the left, and the decimals diminish in the same proportion from the decimal point towards the right.

A number composed partly of a whole number and partly of a decimal, is called a mixed number.

Write the following numbers.

Forty one, and three tenths. 41, 3.
Sixteen, and three millionths. 16, 000003.
Five, and nine hundredths.
Sixty-five, and fifteen thousandths.
Eighty, and three millionths.
Two, and three thousand millionths.
Four hundred and ninety-two thousandths.
Three thousand and twenty-one ten thousandths.
Forty seven, and twenty-one ten thousandths.
Thirty-nine, 3 tenths 4 thousandths 1 millionth.

§ 162. The denominations of Federal Money will correspond to the decimal division, if we regard 1 dollar as the unit. For, the dimes are tenths of the dollar, the cents are hundredths of the dollar, and the mills, being tenths of the cent, are thousandths of the dollar.

Ex. 1. Express $47 and 25 cents decimally.
Ans. $47,25.

2. Express $39,39 cents and 7 mills decimally.
Ans. $39,397.

3. Express $12 and 3 mills decimally.
Ans. $12,003.

ss $147 and 4 cents decimally.
Ans. $147,04.

§ 163. If ciphers be annexed to the numerator of a decimal fraction, the same number of ciphers must also be annexed to the denominator; for there must be as many ciphers in the denominator as there are places of figures in the numerator § 159. The numerator and denominator will therefore be multiplied by the same number, and the value of the fraction will not be changed § 118. Hence,

Annexing ciphers to a decimal fraction does not alter its value.

Ex. 1. The fraction $\frac{6}{10}$ is equal to $\frac{60}{100}=\frac{600}{1000}$; that is, =,6=,60=,600.

§ 164. If ciphers be prefixed to the numerator of a decimal fraction, that is, placed at the left hand of the significant figures, the same number of ciphers must be annexed to the denominator. Now, the numerator will remain unchanged while the denominator will be increased ten times for every cipher which is annexed, and the value of the fraction will be decreased in the same proportion § 116. Hence,

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher so prefixed.

Ex. Take the fraction ,3. Prefixing a cipher, it becomes ,03, prefixing another it becomes ,003. But ,03 is one tenth of ,3, and ,003 is one hundredth of ,3.

QUESTIONS.

§ 158. When the unit 1 is divided into 10 equal parts, what is each part called? What is each part called when it is divided in 100 equal parts? When into 1000, 10,000, &c. How are decimal fractions formed? What gives denomination to the fraction?

§ 159. Are the denominators of decimal fractions generally *set down?* How are the fractions expressed? Is the d nator understood? What is it? What is the place *decimal points* called? The next?—the third? &c.

§ 160. Which way are decimals numerated? Does value of e depend upon the place which it occupie

§ 161. does the value change from the left towa the right do ten parts of any one place make? H may whole numbers be joined with decimals? What is number called when composed partly of whole numbers a partly of decimals?

§ 162. When will the denominations of Federal Money correspond to decimal fractions?

§ 163. If hers are annexed to a decimal, is the value changed? not?

§ 164. If ciphers be prefixed, is the value changed? How much for each cipher ?

ADDITION OF DE AL FRACTIONS.

§ 165. It must be recollected that only like parts of unity can be added together, and therefore in setting down the numbers for addition the figures occupying places of the same value must be placed directly under each other.

The addition of decimal fractions is made in the same manner as that of whole numbers.

Hence we have the following

RULE.

I. *Set down the numbers to be added so that tenths shall fall under tenths, hundredths under hundredths, &c. This will bring all the decimal points directly under each other.*

II. *Then add as in simple numbers and point off in the sum, from the right hand, so many places for decimals as are equal to the greatest number of places in any of the given numbers.*

Ex. 1. Add 37,04, 704,3 and ,0376 together.

In this example the 37,04
of decimal 704,3
sum is 4. ,0376
Sum 741,3776

2. Add 4,035, 763,196, 445,3741 and 91,3754 together.

Ans. 1303, 9805.

3. Add 365,103113, ,76012, 1,34976, ,3549 and 61,11 together.

Ans. 428 ,677893.

4. Required the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty one thousandths.

Ans. 494, 521.

5. Required the sum of two hundred dollars one dime three cents and nine mills, four hundred and forty dollars nine mills, and one dollar one dime and one mill.

Ans. $641 ,249.
or 641 dolls. 2 dimes 4 cents 9 mills.

6. What is the sum of one tenth, one hundredth, and one thousandth.

Ans. ,111.

7. What is the sum of 4 and 6 ten thousandths.

Ans. 4 ,0006.

SUBTRACTION OF DECIMAL FRACTIONS.

§ 166. Subtraction of Decimal Fractions teaches how to find the difference between two decimal numbers.

RULE.

Set down the less number under the greater, so that figures occupying places of the same value shall fall directly under each other : then subtract as in sim numbers and point off the decimal places as in dition.

Ex. 1. From 3,275 take ,0879.

In this example a cipher is annexed to the minuend to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend, § 163.

3 ,2750
,0879
3 ,1871

2. From 3295 take ,0879.

Ans. 3294,9121.

3. From 291,10001 take 41,375.

Ans. 249,72501.

4. From 10,000001 take ,111111.

Ans. 9,888890.

5. From three hundred and ninety-six, take 8 ten thousandths.

Ans. 395,9992.

6. From 1 take one thousandth.

Ans. ,999.

MULTIPLICATION OF DECIMAL FRACTIONS.

Ex. 1. Let it be required to multiply ,37 by ,8.

The two decimals may be expressed $\frac{37}{100}$ and $\frac{8}{10}$: then, $,37\times,8=\frac{37}{100}\times\frac{8}{10}=\frac{296}{1000}=,296$ answer.

2. Multiply ,3 by ,02.

$,3\times,02=\frac{3}{10}\times\frac{02}{100}=\frac{6}{1000}=,006$ answer.

§ 167. It is seen from these examples that the product of the denominators will contain as many ciphers as there are places of decimals in both the numerators. The product, therefore, of the numerators should contain so many places of decimals as there are in both the numbers, and when there are not so many the deficiency must be supplied by pre-
iphers; for otherwise, we could not in the
ject the denominator and express the
cimally. Therefore to multiply one deci-
n by another we have the following

RULE.

Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as are equal to the number of decimal places in the multiplicand and multiplier; and if there be not so many in the product, supply the deficiency by prefixing ciphers.

Ex. 1. Multiply 3,049 by ,012.

Ans. ,036588.

(2.)		(3.)	
Multiply	365 ,491	Multiply	496 ,0135
by	,001	by	1 ,496
Ans.	,365491	*Ans.*	742 ,0361950

4. Multiply one and one millionth by one thousandth.

Ans. ,001000001.

5. Multiply one hundred and forty seven millionths, by one millionth.

Ans. ,000000000147.

6. Multiply three thousand, and twenty seven hundredths by 31.

Ans. 93008 ,37.

NOTE. § 168. When a decimal fraction is to be multiplied by 10, 100, 1000, &c. the multiplication may be made by removing the decimal point as many places to the right hand as there are ciphers in the multiplier, and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

Thus, 6,79 multiplied by $\left\{\begin{array}{l}10\\100\\1000\\10000\\100000\end{array}\right\}$ is $\left\{\begin{array}{l}67{,}9\\679.\\67\\6\\ \\ \end{array}\right.$

DIVISION OF DECIMAL FRACTIONS.

§ 169. Division of Decimal Fractions is similar to that of simple numbers.

We have seen § 47, that when there is no remainder, the divisor multiplied by the quotient will produce the dividend. But when decimal fractions are multiplied together there will be as many decimal places in the product as there are in the multiplier and multiplicand § 167.

Therefore the dividend must contain as many decimal places as the divisor and quotient together. The number of decimal places in the quotient must therefore be equal to the difference between the number of places in the dividend and the number of places in the divisor. Hence, for division of decimal fractions we have the following

RULE.

Divide as in simple numbers, and point off in the quotient, from the right hand, so many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.

Ex. Divide 1,38483 by 60,21.

There are five decimal places in the dividend, and two in the divisor: there must therefore be three places in the quotient: hence one 0 must be prefixed to the 23 and the decimal point placed before it.

```
60,21)1,38483(23
      1 2042
      ------
       18063
       18063
      ------
   Ans. ,023
```

2. 2,3421 by 2,11. Ans. 1,11.

3. Divide 12,82561 by 3,01.
Ans. 4,261.

4. Divide 33,66431 by 1,01.
Ans. 33,331.

5. Divide ,010001 by ,01.
Ans. 1,0001.

6. Divide 8,2470 by ,002.
Ans. 4123,5.

NOTE. § 170. When any decimal number is to be divided by 10, 100, 1000, &c. the division is made by removing the decimal point as many places to the left as there are 0's in the divisor; and if there be not so many figures on the left of the decimal point, the deficiency must be supplied by prefixing ciphers.

$$27{,}69 \text{ divided by} \left\{\begin{array}{l}10\\100\\1000\\10000\end{array}\right\} = \left\{\begin{array}{l}2{,}769\\{,}2769\\{,}02769\\{,}002769\end{array}\right.$$

NOTE 2. § 171. When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers*, § 169.

Ex. 1. Divide 4397,4 by 3,49.

We annex one 0 to the dividend. Had it contained no decimal place we should have annexed two.

```
3,49)4397,40(1260
     349
     ----
      907
      698
     -----
      2094
      2094
     ------
             An
```

2. Divide 2194,02194 by ,100001.

Ans. 21940

3. Divide 9811,0047 by ,325947.

Ans. 30100.

3. Divide ,1 by ,0001.

Ans. 1000.

NOTE 3. 172. When it is necessary to continue the division arther than the figures of the dividend will allow, e may annex ciphers and consider them as decimal id.

Ex. 1.

In this exam one 0 and places in th ceed those i

1,25)4,25(3,4
3,75
500
500
Ans. 3,4

2. Divide ,2 by ,06.

We see in this example that the division will never terminate. In such cases the division should be carried to the third or fourth place, which will give the answer true enough for all practical purposes, and the sign + should then be written, to show that the division may be still continued.

,06),20(3,33+
18
20
18
20
18
20

3. Divide 37,4 by 4,5.

Ans. 8,3111+.

QUESTIONS.

§ 165. What parts of unity can be added together? How
s be set down? How is addition of decimals
you set down the numbers? How do you
ou point off in the sum?

§ 166. What does subtraction of decimals teach ? How do you set down the numbers ? How do you subtract them ? How do you point off in the remainder ?

§ 167. How many decimal places will the product of two decimals contain ? When there are not so many in the product, what do you do ? What is the rule for multiplication of decimals ?

§ 168. How do you multiply a decimal by 10, 100, 1000, &c.

§ 169. What is division of decimals ? How many places of decimals must the dividend contain ? How do you find the number of places in the quotient ? What is the rule for division of decimals ?

§ 170. How do you divide a decimal by 10, 100, 1000, &c.

§ 171. If the decimal places in the divisor exceed those in the dividend what do you do ? What will the figures of the quotient be ?

§ 172. How do you continue the division after you have brought down all the figures of the dividend ? What sign do you place after the quotient ?

APPLICATIONS

IN THE FOUR PRECEDING RULES.

1. A merchant sold 4 parcels of cloth, the first contained 127 and 3 thousandths yards ; the 2d, 6 and 3 tenths yards ; the 3d, 4 and one hundredths yards ; the 4th, 90 and one millionth yards : how many yards did he sell in all ?

Ans. 227,313001*yd.*

2. A merchant buys three chests of tea, the first contains 60 and one thousandth *lb.* ; the second 39 and one ten thousandth *lb.* ; the third, 26 and one tenth *lb.* : how much did he buy in all ?

Ans. 125,1011*lb.*

3. What is the sum of $20 and three hundredths ; $4 and one tenth, $6 and one thousandth, and $18 and one hundredth ?

Ans. $4

4. A puts in trade $504,342; B puts in $350,1965 C puts in $100,11; D puts in $99,334; and E put in 9001,32: what was the whole amount put in?
Ans. $10055,3025.

5. B has $936, and A has $1 three dimes and on mill: how much more money has B than A?
Ans. $934,69,9.

6. A merchant buys 37,5 yards of cloth, at $1,2 per yard: how much does the whole come to?
Ans. $46,87,5.

7. A farmer sells to a merchant 13,12 cords o wood at $4,25 per cord, and 13 bushels of wheat a $1,06 per bushel: he is to take in payment 1 yards of broadcloth at $4,07 per yard, and the re mainder in cash: how much money did he receive
Ans. $16,63.

8. If 12 men had each $339 one dime 9 cents an 3 mills: what would be the total amount of the money?
Ans. $4070,31,6.

9. If one man can remove 5,91 cubic yards earth in a day: how much could 19 men remove?
Ans. 112,29*yd.*

10. What is the cost of 8,3 yards of cloth at $5,4 per yard?
Ans. $45,40,1.

11. If a man earns one dollar and one mill p day: how much will he earn in a year?
Ans. $365,36,5.

12. What will be the cost of three hundred an 75 thousandths of a cord of wood, at $2 per cord?
Ans. $0,75.

13. A man leaves an estate of $1473,194 to divided among 12 heirs: what is each one

Ans. $122,76,8

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

§ 173. The value of every vulgar fraction is equal to the quotient arising from dividing the numerator by the denominator, § 58.

Thus, $\frac{9}{2}=4\frac{1}{2}$. Here the 1 cannot be divided by the 2; but 1 is equal to 10 tenths, equal to 100 hundredths, or equal to 1000 thousandths, &c.: hence we shall have $\frac{9}{2}=4\frac{1}{2}=4\frac{1,0}{2}=4,5$. Therefore, to reduce a vulgar fraction to a decimal we have the following

RULE.

I. *Annex one or more ciphers to the numerator and then divide by the denominator.*

II. *If there is a remainder, annex a cipher or ciphers, and divide again, and continue to annex ciphers and to divide until there is no remainder or until the quotient is sufficiently exact: the number of decimal places to be pointed off in the quotient is the same as the number of ciphers used;—and when there are not so many, ciphers must be prefixed.*

Ex. 1. Reduce $\frac{635}{125}$ to its equivalent decimal.

```
125)635(5,08
    625
    ----
     1000
     1000
     ----
```

Ans. 5,08.

2. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$ to decimals.
Ans. ,25 and ,00797+.

3. Reduce $\frac{12}{480}$, $\frac{27}{39}$, $\frac{3}{1000}$, and $\frac{11}{60000}$ to [illegible]mals.
Ans. ,025; ,692+; ,003; [illegible]

4. Reduce $\frac{1}{}$ and $\frac{5}{}$ to decimals.

5. Reduce [illegible] to a decimal.

Ans. 1,496+,

REDUCTION OF DENOMINATE DECIMA

§ 174. We have seen § 71, that a denon number is one in which the *kind* of unit is de nated or expressed.

A denominate decimal is a decimal fracti which the kind of unit that has been divided pressed. Thus ,5 of a £. and ,67 of a shillin denominate decimals. The unit that was divi the first fraction being £1, and that in the sec shilling.

CASE I.

§ 175. To reduce a denominate number equivalent decimal of a given and higher denc tion.

RULE.

Divide the given number by so many as ma of the next higher denomination, annexing c if necessary. Then divide this quotient by so as make one of the denomination still higher, on, till you reach the denomination to whi number is to be reduced: the last quotient will decimal required.

Ex. 1. Reduce 9*d* to the decimal of a £.

12)9,0

2|0),7|5

£ ,0375 *Ans.* £,03

[illegible]ce 7 drams to the decimal of a lb.

Ans. ,002734

3. Reduce 26*d* to the decimal of a £.
Ans. ,1083333+.

4. Reduce ,056 poles to the decimal of an acre.
Ans. ,00035A.

5. Reduce 14 minutes to the decimal of a day.
Ans. ,0097222*da.*+

6. Reduce ,21 pints to the decimal of a peck.
Ans. ,013125*pk.*

CASE II.

§ 176. To reduce denominate numbers of different denominations to an equivalent decimal of a given denomination.

Ex. Reduce £1 4*s* $9\frac{3}{4}d$ to the denomination of a £. $\frac{3}{4}d = ,75d$, hence, $9\frac{3}{4}d = 9,75d$.
But, $9,75d = 9,75$ of $\frac{1}{12}$ of a shilling$=\frac{9,75}{12}=,8125s$.
Hence, $4s\ 9\frac{3}{4}d = 4,8125s = 4,8125$ of $\frac{1}{20}$ of a £$=$
$\frac{4,8125}{20}=,240625$£. Therefore, £1 4*s* $9\frac{3}{4}d=$
£1,240625.

Hence we have the following

RULE.

Divide the lowest denomination named, by that number which makes one of the denomination next higher, annexing ciphers if necessary: then annex this quotient to the next higher denomination, and divide as before: proceed in the same manner through all the denominations to the last: the last result will be the answer sought.

Ex. 1. Reduce £19 17*s* $3\frac{1}{4}d$ to the decimal of a £.
Ans. £19,863+.

2. *Reduce* 15*s* 6*d* to the decimal of a

3. Reduce 7½*d.* to the denomination of shillings.
Ans. ,625*s.*

4. Reduce 2*lb.* 5*oz.* 12*pwt.* 16*gr.* Troy, to the decimal of a *lb.*
Ans. 2,469444*lb.*+.

CASE III.

§ 177. To find the value of a denominate decimal in terms of integers of inferior denominations.

RULE.

I. *Multiply the given decimal by that number which makes one of the next less denomination, and cut off from the right so many figures as there are places in the given decimal.*

II. *Multiply the figures so cut off by the number which makes one of the next less denomination, and cut off as before, and proceed in the same way to the lowest denomination: the figures to the left will form the answer sought.*

Ex. 1. What is the value of ,832296 of a £.

$$\begin{array}{r} ,832296 \\ 20 \\ \hline 16,645920 \\ 12 \\ \hline 7,751040 \\ 4 \\ \hline 3,004160 \end{array}$$

Ans. 16*s.* 7*d.* 3*far.*

2. What is the value of ,002084*lb.* Troy?
Ans. 12,00384*gr.*

3. What is the value of ,625 of a *cwt.*
Ans. 2qr. 14lb.

4. What is the value of ,625 of a gallon?
Ans. 2qt. 1pt.

5. What is the value of £,3375?

Ans. 6*s*. 9*d*.

6. What is the value of ,3375 of a ton?

Ans. 6*cwt* 3*qr*.

7. What is the value of ,05 of an acre?

Ans. 8P.

QUESTIONS.

§ 173. To what is the value of a vulgar fraction equal? How do you reduce a vulgar fraction to its equivalent decimal?

§ 174. What is a denominate number? What is a denominate decimal?

§ 175. How do you reduce a denominate number to a decimal of a given denomination?

§ 176. How do you reduce denominate numbers of different denominations to decimals of a given denomination?

§ 177. How do you find the value of a denominate decimal in terms of integers of inferior denominations?

REDUCTION OF CURRENCIES.

§ 178. Previous to the act of Congress which established a uniform currency throughout the United States, in dollars, dimes, cents, and mills, each state had its own particular currency. This circumstance has caused the dollar to be divided differently in the different states, though the real value of it is the same in all.

Thus 1 dollar is reckoned in

England	at 4*s* 6*d*,	called	English, or sterling money.
Canada and Nova Scotia	5*s*	,,	Canada Currency.
New York, Ohio, and North Carolina	8*s*	,,	New York Currenc

The New England States, Virginia, Kentucky and Tennessee.	6*s*	,,	New England Currency.
New Jersey, Pennsylvania, Delaware and Maryland.	7*s* 6*d*	,,	Pennsylvania Currency.
South Carolina and Georgia.	4*s* 8*d*	,,	Georgia Currency.

CASE I.

§ 179. To reduce a sum in either currency to Federal Money.

The reduction of these currencies to Federal money consists in this: *having given any sum expressed in pounds, shillings and pence, to find its value in dollars, cents, and mills.*

Take for example the English Currency.

$$£1=20s=240d.$$

$$\$1=4s\ 6d=54d.$$

$$\text{Hence, } £1 : \$1 :: 240 : 54,$$

$$\text{or } \$1=\tfrac{54}{240} \text{ of a } £=\tfrac{9}{40} \text{ of a } £.$$

The number of dollars in any sum will therefore be expressed by the number of times which that sum contains the fraction $\tfrac{9}{40}$.

As similar reasoning may be employed for each of the currencies we have for them all the following

RULE.

I. *Reduce the given sum to the decimal of a £. as in* § 175.

II. *Then find the value of* 1 *dollar in the fraction found and divide the given sum, reduced as above, fraction. The quotient will be the answer*

§ 180. To divide by a fraction we invert the terms and multiply § 156. Then, to find the value of the fraction we divide by the denominator, annexing ciphers if necessary.

Ex. 1. Change £27 3*s* 9*d* to Federal money.

3*s* 9*d*=3,75*s*=,1875 of a £. Hence
£27 3*s* 9*d*=£27,1875.
$1=4*s* 6*d*=54*d*=$\frac{54}{240}$ of a £=$\frac{9}{40}$ of a £.
Then £27,1875÷$\frac{9}{40}$=£27,1875×$\frac{40}{9}$.

```
      27,1875
           40
   9)1087,5000
   $120,83,3+
```

Ans. $120 83cts. 3mills+.

CANADA CURRENCY.

1. Change £25 10*s* 6*d* to Federal money.

£25 10*s* 6*d*=£25,525.
$1=5*s*=60*d*=$\frac{60}{240}$=$\frac{1}{4}$

```
   £25,525
         4
  $102,100
```

Ans. $102 10cents.

2. Change £69 15*s* 5*d* to Federal money.
Ans. $279,08,3+.

NEW ENGLAND CURRENCY.

1. Change £40 6*s* 6*d* to Federal money.

£40 6*s* 6*d*=£40,325.
$1=6*s*=72*d*=$\frac{72}{240}$=$\frac{3}{10}$ of a £.

```
     40,325
         10
   3)403,250
     134,41,6+
```

Ans. $134 41cents 6m

2. Change £125 15*s* 8*d* to Federal money.
Ans. $419 27cts. 7mills+.

NEW YORK CURRENCY.

1. Change £365 10*s* 6*d* to Federal money.
£365 10*s* 6*d*=£365,525
$1=8*s*=$\frac{96}{240}$ of a £=$\frac{4}{10}$ of a £. Then
£365,525
10
4)3655,250
913,81,2+
Ans. $913 81cts. 2mills+.

2. Change £20 18*s* 9*d* to Federal money.
Ans. $52, 34, 3+.

PENNSYLVANIA CURRENCY.

1. Change £9 8*s* 3*d* to Federal money.
£9 8*s* 6*d*=£9,4125.
$1=7*s* 6*d*=90*d*=$\frac{90}{240}$ of a £=$\frac{3}{8}$ of a £.
£9,4125
8
3)75,3000
25,10,00
Ans. $25 10cts.

2. Change £19 18*s* 2*d* to Federal money.
Ans. $53 8cts. 8mills+.

GEORGIA CURRENCY.

1. Change £187 9*s* 10*d* to Federal money.
£187 9*s* 10*d*=£187,49166+
$1=4*s* 8*d*=56*d*=$\frac{56}{240}$ of a £=$\frac{7}{30}$ of a £.
£187,49166
30
7)5624,74980
803,53,5+
Ans. $803,53,5+.

2. Change £26 14*s* 6*d* to Federal money.
Ans. $114,53,5+.

EXAMPLES.

1. Reduce £112 18*s* 9*d* in each of the currencies named, to Federal money.

£112 18*s* 9*d*=£112,9375.

Ans.
£112,9375 Sterling Money =$501,94,4+
£112,9375 Canada Currency =$451,75
£112,9375 New England „ =$376,45,8+
£112,9375 New York „ =$282,34,3+
£112,9375 Pennsylvania „ =$301,16,6+
£112,9375 Georgia „ =$484,01,7+

CASE II.

§ 181. To change any sum expressed in Federal Money to any one of the above currencies.

In changing any one of the currencies to Federal Money, we have divided the given sum by $1 reduced to the fraction of a £. § 179.

Now in passing back from Federal Money to the same currency, we must multiply by the same fraction. Hence, we have the following

RULE.

Multiply the given sum by the divisor before used, belonging to the particular currency; that is, multiply by the numerator and divide by the denominator: the quotient will be the answer in pounds and decimals of a pound: then reduce the decimal part to shillings and pence.

Ex. 1. Reduce $32,78,9 to sterling money.

Here the multiplier will be $\frac{9}{40}$.

```
Hence   $32,789
              9
     40)295,101
      £7,377525
             20
       7,550500
             12
       6,606000
              4
              0
```

Ans. £7 7*s* 6*d* 2*far.*+

EXAMPLES.

1. Reduce $102,85 to the several currencies.

Answer

$102,85=£23 2*s* 9¾*d*+	Sterling Money.
$102,85=£25 14*s* 3*d*	Canada Currency.
$102,85=£30 17*s* 1*d*+	New England Cur.
$102,85=£41 2*s* 9½*d*+	New York Currency.
$102,85=£38 11*s* 4½*d*	Pennsylvania Cur.
$102,85=£23 19*s* 11½*d*+	Georgia Currency.

2. Reduce $250 to the several currencies.

Ans.	$250=£56 5*s*	Sterling Money.
	$250=£62 10*s*	Canada Currency.
	$250=£75	New England Cur.
	$250=£100	New York Currency.
	$250=£93 15*s*	Pennsylvania Cur.
	$250=£58 6*s* 7¾*d*+	Georgia Currency.

The following are the rates at which foreign coins are estimated in the Custom Houses of the United States.

En [illegible] f Congress of 1832 $4,80

Silver Rouble of Russia - - -	$,75
Florin or Guilder of the United Netherl'ds	$,40
Mark Banco of Hamburg - - -	$,33⅓
Real of Plate of Spain - - -	$,10
Real of Vellon of do. - - -	$,05
Milrea of Portugal - - - -	$1,24
Tale of China - - - - -	$1,48
Pagoda of India - - - -	$1,84
Rupee of Bengal - - - -	$,50

QUESTIONS.

§ 178. How was the currency of the United States established? What are its denominations? Is the value of a dollar the same in all the states? How many shillings make a dollar in English Currency?—In Canada Currency?—In New York Currency?—In New England Currency?—In Pennsylvania Currency?—In Georgia Currency?

§ 179. In what does the Reduction of the Currencies to Federal Money consist? Give the rule for Reduction.

§ 180. How do you divide by a fraction?

§ 181. How do you change any sum expressed in Federal Money to any one of the above Currencies?

PRACTICE.

§ 182. Practice is a short method of finding the answers to questions in the Rule of Three, when the first term is unity.

For example, if one yard of cloth cost half a dollar, what will 60 yards cost. This is a question which may be answered by a rule called Practice.

If the cloth had been $1 per yard, the cost of 60 yards would have been $60; but since it is only a part of a dollar per yard, the whole cost will be *same part of* $60, that the cost of one yard i *dollar; that is* ½. Hence the cost is ½ of $60
Ans

§ 183. One number is said to be an aliquot p of another, when it forms an exact part of it: th is, when it is contained in that other an exact nun ber of times.

For example, 25 cents is an aliquot part of a dollar. It is an exact fourth part, and is contained in the dollar four times. So also, 2 months, 3 months, 4 months, and 6 months, are all aliquot parts of a year.

TABLE OF ALIQUOT PARTS.

Cts.	*Parts of $1*	*Mo.*	*Parts of a year.*	*Days*	*Parts of 1 mo*	*Parts of £1.*	*Parts of 1 shilling.*
50	½	6	½	15	½	10*s* = ½	6 *d*= ½
33⅓	⅓	4	⅓	10	⅓	6*s* 8*d*= ⅓	4 *d*= ⅓
25	¼	3	¼	7½	¼	5*s* = ¼	3 *d*= ¼
20	⅕	2	⅙	6	⅕	4*s* = ⅕	2 *d*= ⅙
12½	⅛	1	1/12	5	⅙	3*s* 4*d*= ⅙	1½*d*= ⅛
6¼	1/16		or ⅓ of	3	1/10	2*s* 6*d*= ⅛	1 *d*= 1/12
5	1/20		3 *mo.*			1*s* 8*d*= 1/12	

Ex. 1. What is the cost of 376 yards of cloth at $0,75 per yard?

cts.		$	
50	½	376	
		188	cost at 50cts.
25	¼	94	cost at 25cts.
75	¾	282	cost at ¾doll. or $0,75.

Ans. $282.

Had the cloth cost $1 per yard, the cost of the
3 ould have been $376. Had it cost 50cts.
cost would have been ½ of $376, or
been 25cts. per yard, the cost would
$376 or $94; but the price being
the cost is 188+94=$282.

Ex. 2. What is the cost of 196 yards of cotton, at 9*d* per yard?

196*yd* at 6*d* or ½*s*=98*s*
196*yd* at 3*d* or ¼*s*=49*s*
Therefore, 196*yd* at 9*d* or ¾*s*=147*s*=£7 7*s*. *Ans.*

3. What will 9½ yards of cloth cost at £1 4*s* 6*d* per yard?

9 yards at £1=£9
½ yard at £1= 10*s*
9 yards at 4*s* = 1 16*s*
½ yard at 4*s* = 2*s*
9 yards at 6*d* = 4*s* 6*d*
½ yard at 6*d* = „ 3*d*
Total cost £11·12*s* 9*d*

4. What is the cost of 1000 quills at ¼ cent per quill?

Ans. $2,50cts.

5. What is the cost of 900 lead pencils at 6 cents apiece?

Ans. $54,00.

6. What is the cost of 20*lb* of soap at 6¾cents per pound?

Ans. $1,35.

7. What is the cost of 140 yards of tape at 2¼cts. per yard?

Ans. $3,15.

8. What is the cost of 438 bushels of apples at 31¼cts. per bushel?

Ans. $136,87½.

9. What is the cost of 51½ tons of hay at 12 dolls. *per ton?*

Ans. $618.

10. W is the cost of 231 yards of linen at 7 cents pe rd?

Ans. $173,25.

11. What is the cost of 144*lb* of rice at 3½*d* pe pound?

Ans. £2 2*s.*

12. What is the cost of 14¼ yards of cloth, at 4¾ dolls. per yard?

Ans. $67,68¾.

13. What will 131*lb* ese come to at 1*s* 2*d* per lb.?

Ans. £7 12*s* 10*d.*

14. What will 144 dozen of eggs come to at 1*s* 3*d* per dozen?

Ans. £9.

15. What will 6*gal.* 1*qt.* 1*pt.* 2*gi.* of wine come to, at 5*s* 4*d* per quart.

Ans. £6 17*s* 4*d.*

16. What will 51 acres of land be worth at £3 2*s* 2*d* per acre?

Ans £158 10*s* 6*d.*

17. What will 15*cwt.* 2*qr.* 17*lb.* of sugar come to at 1*s* per pound?

Ans. £87 13*s.*

18. What will 4E. E. 3*qr.* 2*na.* of broadcloth cost at £2 3*s* 8*d* per yard?

Ans. £12 16*s* 6½*d.*

19. What will 1*hhd.* 2*gal.* 3*qt.* 1*pt.* 1*gi.* of molasses come to at 12½cts. per quart?

Ans. $32,95$\frac{5}{16}$.

20. What will be the cost of 27*bu.* 3*pk.* 6*qt.* 1*pt.* *of* 10*s* 2*d* 3*far* per bushel?

Ans. £14 5s 11d 0$\frac{63}{64}$*far.*

SIMPLE INTEREST.

§ 184. Interest is an allowance made for the use of money that is borrowed.

For example, if I borrow $100 of Mr. Wilson, for one year, and agree to pay him $6 for the use of it, the $6 is called the interest of $100 for one year, and at the end of the time Mr. Wilson should receive back his $100 together with the $6 interest, making the sum of $106.

§ 185. The money on which interest is paid, is called the *Principal.*

The money paid for the use of the principal is called the *Interest.*

The principal and interest taken together are called the *Amount.*

In the above example,

$100 is the principal
$ 6 is the interest and
$106 is the amount.

The interest of $100 or £100 for one year, determines the per centage or rate of interest. In the example above the rate of interest is 6 per cent.; or $6 for the hundred. Had $8 been paid for the use of the $100, the rate of interest would have been 8 per cent.; or had $3 only been paid, the rate of interest would have been 3 per cent.

In the New England States, and indeed in most of the other states, the legal interest, that is, the rate of interest established by law, is 6 per cent.

In New York, however, it is 7 and in Louisiana 8 per cent.

CASE I.

§ 186. To find the interest on any given principal for one or more years.

RULE.

I. *Multiply the principal by the rate of interest, and divide the product by* 100 *: the quotient will be the interest for one year.*

II. *When the number of years exceeds one, multiply the interest for one year by the number of years: the product will be the interest for that number of years.*

Ex. 1. What is the interest of $650 for one year, at 6 per cent ?

$100 : $6 : : $650 : $39,00 *Ans.*

It is plain, that $100 is to $6, its interest for one year, as $650 to its interest for the same time. The fourth term is found by multiplying the second and third terms together and dividing by the first, § 109. Hence the reason of the rule is evident.

2. What is the interest on $950 for four years at 7 per cent. perann. ?

$950
7
$66,50 Interest for 1 year
4 number of years.
$266,00

Ans. $266.

3. What is the interest of $327,45 for one year, at 6 per cent. perann. ?

We first multiply the given sum $327,45 by the rate of interest, 6 per cent., leaving two places *for decimals* or cents at *the right hand.* Then to *divide by* 100 we remove the decimal point two

$327,45
6
100)1964,70
19,64,70
Ans. 19 dolls. 64 cts. 7 m.

places to the left, § 170, leaving four places for decimals to the right hand. If the principal had contained mills, there would have been five places of decimals in the answer.

§ 187. Hence we have the following

RULE.

I. *When the principal contains dollars only, multiply by the rate of interest, and point off two places from the right hand for cents: the places on the left will express the dollars.*

II. *When the principal contains dollars and cents, multiply as before, and strike off four places from the right hand for cents and mills.*

III. *When the principal contains dollars, cents, and mills, multiply as before and strike off in the product five places from the right hand: the places to the left will be dollars.*

Ex. 4. What is the interest on $375, 27 *cts.* 3 *mills* for two years, at 7 per cent. perann. ?

$375,27,3
7
$26,26,9,11 Interest for one year.
2
$52,53,8,22

Ans. $52,53+.

5. What is the interest on $211,26 for one year at 4½ per cent. ?

$211,26
4½ per cent.
84504
10563 ½ per cent.
9,50,67

Ans.

6. What is the interest on $1576,91 for 3 ye
at 7 per cent.?

Ans. $331,15+.

7. What is the interest on $957,08 for 6 *years a*
3½ per cent.?

Ans. $200,98+.

CASE II.

§ 188. To find the interest at 6 per cent. per annum for any number of months.

At six per cent. per annum, each month produces ½ per cent. on the principal; and every *two months* produces one per cent. on the principal. Therefore, to find the interest for months we have the following

RULE.

Multiply the principal by half the number of months, and remove the separating point in the product two places farther to the left hand.

1. What is the interest of $327 for 8 months at 6 per cent. per annum?

$327

4 half the number of months.

Ans. $13,08

2. What is the interest of $327,47 for 9 months at 6 per cent. per annum?

$327,47

4½ half the months.

130988

16373½

14,73,61½ *Ans.* $14,73+.

CASE III.

find the interest at 6 per cent. per an-
number of days.

Since in computing interest the month is always reckoned at 30 days, 60 days, which make 2 months, will give an interest of one per cent. on the principal. Now, if the interest at one per cent. (which is the principal with the separating point removed two places to the left,) be divided by 60, the quotient will be the interest for 1 day, and this quotient multiplied by the number of days will give the interest required. To divide by 60 we remove the separating point one place farther to the left, and then divide by 6.

Hence we have the following

RULE.

I. *Remove the separating point in the principal three places to the left, then divide by* 6 *and the quotiet will be the interest for* 1 *day.*

II. *Multiply the interest by the number of days, and the product will be the answer sought.*

Ex. 1. What is the interest on $327,30 for 25 days?

6),32730

,05455 Interest for one day.

25 number of days.

27275

10910

$1,36,375 *Ans.* $1,36+.

2. What is the interest on $27,25 for 19 days?

6),02725

,00454+

19

04086

00454

$0,08,626

Ans. ,0

When there are not three figures in the principal, on the left of the separating point, ciphers must be prefixed to *supply* the deficiency.

3. What is the interest of $575,72 for 29 days?
Ans. $2,78+.

4. What is the interest of $195,19 for 7 days?
Ans. $0,22+.

CASE IV.

§ 190. To find the interest at 6 per cent. per annum for years, months, and days.

RULE.

Find the interest for the years by Case I., *for the months by Case* II., *and for the days by Case* III.; *then add the several results together, and their sum will be the answer sought.*

Ex. 1. What is the interest on $1597,27 at 6 per cent. for 3 years 9 months and 11 days?

$1597,27	$1597,27	6)1,59727
6	4½	,26621+
95,83,62	638908	11
3 *yr.*	79863½	2,92831
$287,50,86	$71,87,71½	

Interest for 3 years	$287,50,8+
. . . . . . 9 months	71,87,7+
. . . . . . 11 days	2,92,8+
Total interest	$362,31,3

2. What is the interest of $11759,10 at 6 per cent. for 9 years 11 months and 16 days?
Ans. $7028,02+.

What is the interest on $9787 for 12 years and

Ans. $7048,27+.

4. What is the interest of $87601,29 for 1 year 1 month and 1 day? *Ans.* $5708,68+.

5. What is the interest of $806,90, for 1 year and 10 months at 6 per cent. per annum? *Ans.* $88,75+.

6. What is the interest of $450,75 for 4 years and 7 months, at 6 per cent. per annum? *Ans.* $123,95+.

7. What is the interest of $443,50 for 7 years 2 months and 12 days, at 6 per cent. per annum? *Ans.* $191,59+.

8. What will be the total amount of $649,22 after 10 years and 10 months, at an interest of 6 per cent? *Ans.* $1071,21+.

CASE V.

§ 191. When there are months and days, and the rate of interest is greater or less than 6 per cent.

RULE.

Find the interest as at 6 per cent. Then add to it or subtract from it such a part of the interest so found as the given rate exceeds or falls short of six per cent. per annum.

Ex. 1. What is the interest of $119,50 at 7 per cent. per annum, for 3 years and 4 months?

119,50 119,50
2 half the months 6
$2,39,00 int. for 4 mo. 7,17,00
3
21,51,00 int. for 3 y
2,39,00 int. for 4

Total interest at 6 per cent. $23,90,00
Add one sixth 3,98,33+
Total interest at 7 per ct. $27,88,33+

2. What is the interest on $487,25 for 4 years and 9 months, at 4 per cent. ?

$487,25		$487,25	
4½	half months	6	
194900		$29,23,50	
24362+		4	years
$21,92,62	int. for 9 mo.	116,94,00	int. for 4 yr.
		21,92,62	int. for 9 mo.
Total interest at 6 per ct.		$138,86,62	
Subtract one third		46,28,87+	
Total interest at 4 per ct.		$92,57,75+	

3. What is the interest of $987,99, at 5 per cent. for 5 years 2 months and 9 days ?

Ans. $256,46+.

4. What is the interest on $437,21, at 3 per cent. for 9 years and 9 months ?

Ans. $127,88+.

5. What is the interest of $15000 for 8 months, at 7 per cent. per annum ?

Ans. $700.

6. What is the interest of $400 for 21 days, at 5 per cent. per annum ?

Ans. $1,16+.

7. What is the interest of $876,48, at 7 per cent. for 4 years, 9 months, and 14 days ?

Ans. 293,81,5+.

8. What will be the total amount of $1119,48 after 2 years and a half, at an interest of 7 per cent. per annum ?

Ans. 1315,38,9+.

In computing interest, it is often very to find the interest for the months by sidering them as parts of a year, and the interest for the days by considering them as parts of a month.

Ex. What is the interest of $806,90 for one year, 10 months, and 10 days, at 6 per cent.?

$806,90

6

6)$48,41,40 interest for one year.

2)$8,06,9 int. for 2 mo. $8,06,9

3)4,03,4+ int. for 1 mo. 5

1,34,4+ int. for 10 da. 40,34,5 int. for 10mo

Interest for 1 year, $48,41,40

" " 10 months 40,34,5

" " 10 days 1,34,4+

Total interest 90,10,3+.

2. What is the interest of $200 for 10 years 3 months and 6 days, at 7 per cent.?

$200

7

4)$14,00 int. for 1 year. $14,00

3)$3,50 int. for 3 mo. 10

5)1,16+ int. for 1 mo. $140,00 for 10 yrs.

,23+ int. for 6 days.

$140,00 int. for 10 years.

3,50 int. for 3 months.

,23+ int. for 6 days.

Ans. $143,73+.

3. What is the interest of $132,26 for 1 year 4 months 10 days, at 6 per cent.?

Ans. $10,80+

4. What is the interest of $25,50 for 1[illegible] months and 12 days, at 6 per cent.?

Ans. $2,[illegible]

CASE VI.

§ 192. When the sum ón which the interest is to be cast is in pounds, shillings, and pence.

RULE.

I. *Reduce the shillings and pence to the decimal of a pound*, § 176.

II. *Then find the interest as though the sum were dollars and cents; after which reduce the decimal part of the answer to shillings and pence*, § 177.

Ex. 1. What is the interest, at 6 per cent. of £27 15*s* 9*d* for 2 years?

```
£27  15s  9d=£27,7875
                    6
             --------
             1,667250
                    2
             --------
            £3,334500
                   20
             --------
             6,690000
                   12
             --------
             8,280000
                    4
             --------
             1,120000
```

Ans. £3 6*s* 8¼*d*+.

2. What is the interest on £67 19*s* 6*d* at 6 per cent. for 3 years 8 months 16 days?

Ans. £15 2*s* 8¼*d*.+

3. What is the interest on £127 15*s* 4*d* at 6 per cent. for 3 years and 3 months?

Ans. £24 18*s* 3¼*d*.+ .

4. *What is* the interest of £107 16s 10d at 6 per *for 3* years 6 months and 6 days?

Ans. £22 15s 1d+.

5. What will £279 13*s* 8*d* amount to in 3 years and a half, at 5¼ per cent. per annum?

Ans. £331 1*s* 6*d*+.

6. What is the interest of £514 10*s* 2*d* for 3 years and a half, at 4 per cent.?

Ans. £72 0*s* 7*d*+.

7. What is the interest of £523 11*s* 6*d* for 3 years and a half, at 6 per cent.?

Ans. £109 19*s* 0*d*+.

APPLICATIONS.

$127,50

1. For value received I promise to pay on the 10th day of June next, to Wm. Johnson or order, the sum of one hundred and twenty-seven dollars and fifty cents with interest from date, at 7 per cent.

New York, Jany. 1st, 1833. *John Liberal.*

How much must be paid on the 10th of June?

Ans. $131,46+.

2. For value received I promise to pay on the 4th of July, 1835, to Wm. Johnson or order three hundred and six dollars with interest at 6 per cent. from the 1st of March, 1833.

New York, Jany. 1st, 1833. *John Liberal.*

$306

How much must be paid on this note when it becomes due?

Ans. $349,04+.

We shall now give the rule established in New-York, (See Johnson's Chancery Reports, Vol. I. *page* 17,) for computing the interest on *note,* when partial payments have been same rule is also adopted in Massachusetts, and in most of the other states.

RULE.

I. *Compute the interest on the principal to the time of the first payment, and if the payment equal this interest, add the interest to the principal and from the sum subtract the payment: the remainder forms a new principal.*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all, shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal on which interest is to be computed as before.*

Ex. 1. *For value received I promise to pay* James Wilson, *or order three hundred and fifty-nine dollars ninety-nine cents and eight mills with interest, at 6 per cent.*

May 1st, 1826. James Paywell.

$359.99,8.

On this note were endorsed the following payments

Dec. 25th. 1826 Received $39.99,8
July 10th. 1827 „ $ 4.99,8
Sept. 1st. 1828 „ $15.00,0
June 14th. 1829 „ $99.99,9

What was due April 15th. 1830?

Ans. $302.16,1+.

Principal on int. from May 1st 1826 ... $359.99,8
Interest to Dec. 25th. 1826, time of
it, 7 months 24 days 14.03,9+
Amount 374.03,7

Payment Dec. 25th, exceeding int.		$363,64,7
then due, ...		$ 49,99,8
Remainder for a new principal		$313,64,9
Interest of $313,64,9 from Dec. 25th, 1826, to June 14th, 1829, 2 years 5 months 19 days, ...		$ 46,47,2+
Amount		$360,12,1
Payment, July 10th, 1827, less than interest then due.	$ 4,99,8	
Payment, Sept. 1st. 1828.	15,00,8	
Their sum . . . . less than int. then due	$ 20,00,6	
Payment June 14th, 1829.	99,99,9	
Their sum exceeds the interest then due		$120,00,5
Remainder for a new Principal June 14th, 1829 . . .		$240,11,6
Interest of $240,11,6 from June 14th 1829, to April 15th 1830, 10 months 1 day		12,04,5
Total due, April 15th, 1830.		$252,16,1+

2. *For value received, I promise to pay* William Jenks, *or order, three thousand four hundred and sixty nine dollars and thirty two cents, with interest from date, at* 6 *per cent. Feb. 6th,* 1825.

$3469,32 Bill Spendthrift.

On this note were endorsed the following payments:

May 16th 1828, received	$545,76
May 16th 1830, received	$1276
Feb. 1st 1831, received	$2074,72

What remained due Aug. 11th, 1832?

Ans. $860,91+

3. *A's note* of $635,84 was dated Sept. 5th

COMPOUND INTEREST.

§ 193. Compound Interest is when the inte
a sum of money becoming due, and not being
is added to the principal, and the interest the
culated on this amount, as on a new principal.
example, suppose I were to borrow of Mr. W
$200 for one year, at 6 per cent. and at the
the year pay him neither the interest nor prin
Now if Mr. Wilson should add the interest $
the principal $200, making $212, and charg
with interest on this sum till I paid him, this
be Compound Interest, because it is interest
interest.

RULE.

*Calculate the interest to the time at which
comes due: then add it to the principal and
late the interest on the amount as on a new p
pal: add the interest again to the principal
calculate the interest as before: do the same f
the times at which payments of interest become
from the last result subtract the principal, an
remainder will be the compound interest.*

Ex. 1. What will be the compound interest
per cent. of $3750 for 4 years, the interest bein

	$3750,00	prin. for first yr.
$3750×7÷100=	262,50	int. for 1st year.
	4012,50	principal 2d „
$4012,50×7÷100=	280,87+	int. for 2d „
	4293,37+	prin. 3d „
$4293,37×7÷100=	300,53+	int. 3d „
	4593,90+	prin. 4th „
$4593,90×7÷100=	321,57+	int. 4th „
	4915,47+	amt. at 4 yrs.
1st principal	3750,00	
	$1165,47+	amt. of int.

2. What will be the compound interest on $295,37 at 6 per cent. for 2 years, the interest being added annually?

Ans. $36,50+.

3. What will be the compound interest on $500 for one year at 8 per cent. the interest being computed quarterly?

Ans. $41,21+.

QUESTIONS.

§ 184. What is Interest?

§ 185. What is the money called on which interest is paid? What is the money called which is paid for the use of the principal? What is the amount? What determines the rate of interest? What is legal interest? How much is it in most of the states? What is it in New York?

§ 186. How do you find the interest on any principal for one or more years?

§ 187. If the principal contains only dollars, how do you find the interest? When it contains dollars and cents, what do you do? How do you do when it contains dollars, cents, and mills?

§ 188. How do you find the interest for any months at 6 per cent.?

§ 189. How do you find the interest for any number of days at 6 per cent.?

§ 190. How do you find the interest at 6 per cent. for years, months, and days?

§ 191. How do you do when there are months and days, and the rate of interest is greater or less than 6 per cent.?

§ 192. How do you find the interest when the sum is in pounds, shillings, and pence?

§ 193. What is compound interest? How do you calculate compound interest?

COMMISSION AND BROKERAGE.

§ 194. Commission and Brokerage are allowances made to Factors and Brokers for their services in buying and selling. The allowance made is generally a certain per cent. on the moneys paid out or received, and the amount may be determined by the rules of simple interest.

Ex. 1. What is the commission on $4396 at 6 per cent.?

$$\begin{array}{r} 4396 \\ 6 \\ \hline 263{,}76 \end{array}$$

Ans. $263,76.

2. A factor sells 60 bales of cotton at $425 per bale, and is to receive 2½ per cent. commission: how much must he pay over to his principal?

Ans. $24862,50.

3. A sent to B, a broker, $3825 to be invested in [illegible] to receive 2 per cent. on the amount [illegible]ock: what was the value of the stock

As B is to receive 2 per ct., $102 of A's money will purchase $100 of stock: hence 100+ the commission, is to 100, as the given sum to the stock which it will purchase.

100
2
102 : 100 : : 3825
100
102)382500(3750
306
765
714
510
510

Ans. $3750.

PROOF.

$3750
Commission on $3750 at 2 pr. ct. = 75
Total Sum $3825

4. A factor receives $708,75, and is directed to purchase iron at $45 per ton: he is to receive 5 per cent. on the money paid: how much iron can he purchase? *Ans.* 15*tons.*

INSURANCE.

§ 195. Insurance is an agreement by which an individual or a company agrees to exempt the owners of certain property from loss or hazard.

The written agreement is called the *policy.*

The premium is the amount paid by him who owns the property, to those who insure it, as a compensation for their risk. It is generally so much *per cent.* on the value of the property insured.

Ex. 1. What would be the premium for the *rance of* a house valued at $5500 against loss *for 1 year,* at ½ per cent?

By removing the separatrix two places to the left we obtain $55,00 the insurance at 1 *per cent.* $27,50 is the insurance at ½ per cent.

$55,00 at 1
$27,50 at ½
Half

2. What would be the premium for *insuring* ship and cargo, valued at $37500 from *New* Y to Liverpool, at 3½ per cent.?

Ans. 1312,50

3. What would be the insurance on a ship valued at $47520 at ½ per cent.: also at ⅓ per cent.?

Ans. $237,60.—$158,40.

DISCOUNT.

§ 196. If I give my note to Mr. Wilson for $106, payable in one year, the present value of the note will be less than $106 by the interest on its present value for one year: that is, its present value will be $100.

The amount named in a note is called the *face of the note.* Thus $106 is the face of the note to Mr. Wilson.

The present value of a note is that sum which being put at interest until the note becomes due would increase to an amount equal to the face of the note. Thus $100 is the present value of the note to Mr. Wilson.

The discount is the difference between the face of a note and its present value. Thus, $6 is the discount on the note to Mr. Wilson.

RULE.

As 100 + interest of $100 for the given time, is to 100, so is the face of the note to its present value.

Ex. 1. What is the present value of a note for $1828,75 due in one year, at 4½ per cent. per annum?

100

4,50 Int. of $100 for the time.

104,50 : 100 :: 1828,75

100

104,50)182875,00($1750

Ans. $1750.

2. What is the present value of a note for $1290,81 discounted for four months, at 6 per cent. per annum? *Ans.* $1265,50.

5. What is the present value of $800, due 4 years hence: the interest being computed at 5 per cent. per annum? *Ans.* $666,66,6+.

NOTE.—When payments are to be made at different times, *find the present value of the several sums separately and their sum will be the present value of the note.*

4. What is the present value of a note for $3500 on which $300 are to be paid in 6 months; $900 in one year; $1300 in eighteen months; and the residue at the expiration of two years: the rate of interest being 6 per cent. per annum?

Ans. $3225,44+.

5. What is the discount of £1500 one half payable in 6 months and the other half at the expiration of a year, at 7 per cent. per annum?

Ans. £74 8*s* 6¾*d*+.

6. What is the present value of $2880, one half payable in 3 months, one third in 6 months, and the rest in 9 months, at 6 per cent. per annum?

Ans. $2810,08+

LOSS AND GAIN.

§ 197. Loss and Gain is a rule by which chants discover the amount lost or gained in the chase and sale of goods. It also instructs them much to increase or diminish the price of their so as to make or lose so much per cent.

Ex. 1. Bought a piece of cloth containing 75yd at $5,25 p t at $5,75 cents per yard: how n the trade?

of 1 yard.
of 1 yard.
rofit on 1 yard.

yd *yd* cts.
1 : 75 : : 50
75
$37,50 *Ans.* $37,50.

2. Bought a piece of calico containing 50*yd.* at 2*s* 6*d* per yard: what must it be sold for per yard to gain £1 0*s* 10*d*?

50*yd.* at 2*s* 6*d*=£6 5*s*
Profit =£1 0 10*d*
It must sell for £7 5*s* 10*d*
50)£7 5*s* 10*d*(2*s* 11*d*
Ans. 2*s* 11*d*.

3. Bought a hogshead of brandy at $1,25 per gallon, and sold it for $78: was there a loss or gain?
Ans. loss of $0,75.

4. A merchant purchased 3275 bush wheat for which he paid $3517,10, but findi o *is willing* to lose 10 per cent.: what
l?

Ans.

5. A. bought a piece of cotton containing 40 yards at 6 cents per yard; he sold it for $7\frac{1}{2}$ cents per yard: how much did he gain?

Ans. $0,60.

6. Bought a piece of cloth containing 75 yards for $375: what must it be sold for per yard, in order to gain $100?

Ans. $6,33$\frac{1}{3}$ per yard.

7. Bought a quantity of wine at $1,25 per gallon, but it proves to be bad and am obliged to sell it at 20 per cent less than I gave: how much must I sell it for per gallon?

Ans. $1 per gall.

8. A farmer sells 125 bushels of corn for 75cts. per bushel; the purchaser sells it at an advance of 20 per cent.: how much did he receive for the corn?

Ans. $112,50.

9. A merchant buys one tun of wine for which he pays $725, and wishes to sell it by the hogshead at an advance of 15 per cent.: what must he charge per hogshead?

Ans. $208,43+.

10. A merchant buys 158 yards of calico for which he pays 20 cents per yard; one half is so damaged that he is obliged to sell it at a loss of 6 per cent.; the remainder he sells at an advance of 19 per cent.: how much did he gain?

Ans. $2,05+.

TARE AND TRET.

§ 198. *Tare* and *Tret* are allowances made in selling goods by weight.

Draft is an allowance on the gross weight in fa-vour of the buyer or importer: it is always dedu before the *Tare*.

Tare is an allowance made to the buyer for weight of the hogshead, barrel or bag, &c. containing the commodity sold.

Gross Weight is the whole weight of the goods together with that of the hogshead, barrel, bag, &c. which contains them.

Suttle is what remains after a *part of* the allowances have been deducted from the gross weight.

Net Weight is what remains after all the deductions are made.

EXAMPLES.

1. What is the net weight of 25 hogsheads of sugar, the gross weight being 66*cwt.* 3*qr.* 14*lb.*: tare 11*lb.* per hogshead?

	cwt.	*qr.*	*lb.*	
	66	3	14	gross
25×11=275*lb.*	2	1	23	tare
Ans.	64	1	19	net.

2. If the tare be 4*lb.* per hundred, what will be the tare on 6*T.* 2*cwt.* 3*qr.* 14*lb.*

Tare for 6*T.* or 120*cwt.*=480*lb.*
2*cwt.* = 8
3*qr.* = 3
14*lb.* = 0½
Tare 491½
Ans. 4*cwt.* 1*qr.* 15½*lb.*

3. What is the net weight of 32 boxes of soap, weighing 31550*lb.* allowing 4*lb.* per box for draft and 12 per cent. for tare.

31550 gross　　31422
1= 128　　12
31422　　3770,64
54*lb.*=1T. 13cwt. 2qr. 18lb. 10oz.+

4. What will be the cost of 3 hogsheads of tobacco at $9,47 per cwt. net: the gross weight being of

	cwt.	*qr.*	*lb.*		*lb.*
No. 1.	9	3	25	tare	146
„ 2.	10	2	12	tare	150
„ 3.	11	1	25	tare	158

Ans. $265,16.

5. At £1 5*s* per *cwt.* net, what will be the cost of 4 hogsheads of sugar weighing gross,

	cwt.	*qr.*	*lb.*
No. 1.	10	3	6
„ 2.	12	5	19
„ 3.	13	1	10
„ 4.	11	2	7
	49	0	14 gross
Tare 4lb. per cwt.	1	3	0 8oz.
	47	1	13 8oz. net.

Ans. £59 4*s* 3*d*+.

6. At 21 cents per *lb.* what will be the cost of 5*hhd.* of coffee, weighing in gross,

	cwt.	*qr.*	*lb.*	*tare,*
No. 1.	6	2	14	94
„ 2.	9	1	20	100
„ 3.	6	2	22	88
„ 4.	7	2	25	89
„ 5.	8	0	13	100

Ans. $808,71.

7. At £7 5*s* per cwt. net, how much will 16*hhd.* of sugar come to, each weighing gross 8cwt. 3qr. 7*lb.*: tare 12*lb.* per cwt.?

Ans. £912

QUESTIONS.

§ 194. What are Commission and Brokerage
allowance generally made?

§ 195. What is Insurance? What is the
ment called? What is the premium? How
determined?

§ 196. What is the face of a note? What
value of a note? What is the discount of a
you determine the present value?

§ 197. What is the use of the rule of Loss and
what else does it instruct him?

§ 198. What are Tare and Tret? What is D
is Tare? What is Gross Weight? What is Sutt
Net Weight?

FELLOWSHIP.

§ 199. Fellowship is the joining together
persons in trade with an agreement to share
and profits according to the amount which
puts into the partnership. The money em
called the *Capital Stock.*

The gain or loss to be shared is called
dend.

It is plain that the whole stock which st
gain or loss, must be to gain or loss, as the
any individual, to his part of the gain or loss
we have the following

RULE.

As the whole stock is to the whole gain
is each man's share, to his share of the gain

EXAMPLES.

B buy certain merchandise
which A pays £90, and B

gain by the purchase £32: what is each one's share of the profits?

A . . £90
B . . £70
£160 : 32 : : { 90 : £18 A's share.
                   70 : £14 B's share.

2. A and B have a joint stock of $2100, of which A owns $1800 and B $300: they gain in a year $1000: what is each one's share of the profits?

Ans. { A's share $857,14+.
           B's share $142,85+.

3. A, B, C and D have £20,000 in trade; at the end of a year their profits amount to £16,000: what is each one's share, supposing A to receive £50 and D £30 for extra services out of the profits?

Ans. { A's share £4030.
           B's share £3980.
           C's share £3980.
           D's share £4010.

4. Five persons A, B, C, D and E have to share between them an estate of $10,000: A is to have one fourth; B one eighth; C one sixth; D one eighth; and E what is left: what will be the share of each?

Ans. { A's share $2500.
           B's „ $1250.
           C's „ $1666,66+
           D's „ $1250.
           E's „ $3333,33+.

DOUBLE FELLOWSHIP.

§ 200. When several persons who ar
gether in trade employ their capital for

riods of time, the partnership is calle
lowship.

For example, suppose A puts $100
years: B $200 for 2 years: and C
year: this would make a case of doul
-- Now it is plain that there are two
which should determine each one's sh
fits: 1st. *the amount of capital he puts*
the time which it is continued in the b

Hence each one's share should be
the capital he puts in, multiplied by
continued in trade. Therefore we hav

RULE.

Multiply each man's stock by the tin
it in trade: then say, as the sum of the
the whole gain or loss, so is each part
to each man's share of the gain or loss

EXAMPLES.

1. A and B enter into partnershi
£840 for 4 months, and B puts in
months; they gain £300: what is ea
of the profits?

A's stock - - - 840×4=3360
B's stock - - - 650×6=3900

7260 : 300 :: { 3360
3900

2. A put in trade £50 for 4 mont
for 5 months; they gained £24: how
vided between them?

Ans. { A's sha
B's sha

hold a pasture toge
C pastures 23 hor

and D 21 horses for 39 days: how much of the rent ought each one to pay?

Ans. { C must pay £23 5*s* 9*d*.
D must pay £30 14*s* 3*d*.

EQUATION OF PAYMENTS.

§ 201. I owe Mr. Wilson $2 to be paid in 6 months; $3 to be paid in 8 months; and $1 to be paid in 12 months. I wish to pay his entire dues at a single payment, to be made at such a time, that neither he nor I shall lose interest: at what time must the payment be made?

The method of finding the mean time of payment of several sums due at different times, is called *Equation of Payments.*

Taking the example above.

Int. of $2 for 6mo.	=int. of $1 for 12mo.	2×6=12
„ of $3 for 8mo.	=int. of $1 for 24mo.	3×8=24
„ of $1 for 12mo.	=int. of $1 for 12mo.	1×12=12
6	48	48

The interest on all the sums, to the times of payment, is equal to the interest on $1 for 48 months; and 48 is equal to the sum of all the products which arise from multiplying each sum of money by the time before which it becomes due. But the sum of the payments is equal to 6. Now if $1 will produce a certain interest in 48 months, what time will be necessary for $6 to produce the same interest? Obviously but one sixth of the time, or 8 months. Hence we have the following

RULE.

Multiply each payment by
comes due, and divide the sum

sum of the payments: the quotient will be [illegible] time.

2. B owes A $600: $200 is to be paid in two months, $200 in four months, and $200 in six months: what is the mean time for the payment of the whole?

200×2= 400
200×4= 800
200×6=1200
600)24|00
4 *Ans.* 4 *months.*

3. A merchant owes $600, of which $100 is to be paid in 4 months, $200 in 10 months, and the remainder in 16 months: if he pay the whole at once, at what time must he make the payment?

Ans. 12 *months.*

4. A owes B $600; one third is to be paid in 6 months, one fourth in 8 months, and the remainder in 12 months: what is the mean time of payment?

Ans. 9 *months.*

5. A merchant has due him $300 to be paid in 60 days, $500 to be paid in 120 days, and $750 to be paid in 180 days: what is the equated time for the payment of the whole?

Ans. 137$\frac{13}{31}$ *days.*

6. A merchant has due him $1500; one sixth is to be paid in 2 months; one third in 3 months; and the rest in 6 months: what is the equated time for the payment of the whole?

Ans. 4⅓ *months.*

QUESTIONS.

is Fellowship? What is the gain or loss [illegible]
rule for finding each one's share?
is Double Fellowship? What was [illegible]

stances determine each one's share of the profits? What is the rule for finding each one's share?

§ 201. What is Equation of Payments? What is the rule for finding the mean time of payment of several sums due at different times?

DUODECIMALS.

§ 202. Duodecimals are denominate fractions in which 1 foot is the unit that is divided.

The unit 1 foot is first supposed to be divided into 12 equal parts, called inches or primes, and marked ′.

Each of these parts is supposed to be again divided into 12 equal parts, called seconds, and marked ″.

Each second is divided in like manner, into 12 equal parts called thirds, and marked ‴.

This division of the foot, gives

1′ inch or prime - - $=\frac{1}{12}$ of a foot
1″ second is $=\frac{1}{12}$ of $\frac{1}{12}$ - $=\frac{1}{144}$ of a foot.
1‴ third is $=\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - $=\frac{1}{1728}$ of a foot.

Duodecimals are added and subtracted like other denominate numbers, 12 of a lesser denomination making one of a greater, as in the following

TABLE.

12‴	make	1″ second
12″	- -	1′ inch or prime
12′	- -	1 foot.

Ex. 1. In 185′, how many feet?

Ans. 15*ft.* 5′.

2. In 250″, how many feet and inches?

Ans. 1ft. 8' 10'.

3. In 4367‴, how many feet?

Ans. 2ft. 6' 3''

EXAMPLES IN ADDITION AND SUBTRAC

1. What is the sum of 3*ft.* 6′ 3″ 2‴ and
10″ 11‴? *Ans.* 5*ft.* 8′ 2″ 1

2. What is the sum of 8*ft.* 9′ 7″ and 6*ft.* 7′
4‴? *Ans.* 15*ft.* 4′ 10″ 4‴.

3. What is the difference between 9*ft.* 3′ 6″
and 7*ft.* 3′ 6″ 7‴? *Ans.* 1*ft.* 11′ 10″ 11‴.

4. What is the difference between 40*ft.* 6′ 6″
29*ft.* 7‴? *Ans.* 11*ft.* 6′ 5″ 5‴.

MULTIPLICATION OF DUODECIMALS.

§ 203. It has been shown § 84, that feet mul plied by feet gives square feet in the product.

Ex. 1. Multiply 6*ft.* 6′ 6″ by 2*ft.* 7′.

Set down the multiplier under the multiplicand, so that feet shall fall under feet, inches under inches, &c. It is generally most convenient to begin with the highest denomination of the multiplier, and first multiply the lower denominations of the multiplicand.

ft.
6 6′ 6″
2 7′
13 1′
3 9′ 9″ 6‴
16 10′ 9″ 6‴

The 6″ of the multiplicand, are $\frac{6}{12}$ of an inch, o $\frac{6}{144}$ of a foot. Therefore when we multiply it by feet, the product is 12″, equal to 1 inch. Multiplying 6′, or $\frac{6}{12}$ of a foot, by 2 feet the product is 12′, which add 1 inch from the last product, making 13′. Set down 1′ under the column of inches and carry 1 foot to the product of the 6 by 2, making 13 feet.

Then multiply by 7′. The product of 7′ by 6″= 42‴: for 7′=$\frac{7}{12}$ of a foot, and 6″=$\frac{6}{144}$ of a foot: hence 7′ × 6″=$\frac{7}{12}$ × $\frac{6}{144}$=$\frac{42}{1728}$=42‴=3″ 6‴. Then $\frac{7}{12}$ × $\frac{6}{12}$=$\frac{42}{144}$=42″ and 3″ to carry make 45″=3′ 9″

set d Then $\frac{7}{12}$ by 6=42′ and 3′ to carry ma

are set down in their proper pla

§ 204. Hence we see

1st. *That feet multiplied by feet gives square feet in the product.*

2d. *That feet multiplied by inches gives inches in the product.*

3d. *That inches multiplied by inches gives seconds, or twelfths of inches in the product.*

4th. *That inches multiplied by seconds gives thirds in the product.*

2. Multiply 9*ft.* 4*in.* by 8*ft.* 3*in.*

Beginning with the 8 feet, we say 8 times 4 are 32′, which are equal to 2 feet 8′: sent down the 8′. Then say 8 times 9 are 72 and 2 to carry are 74 feet: then multiplying by 3′, we say, 3 times 4′ are 12″, equal to 1 inch: set down 0 in the seconds place: then 3 times 9 are 27 and 1 to carry make 28′, equal to 2*ft.* 4′. Therefore the entire product is equal to 77*ft.*

$$\begin{array}{rrrl}
9 & 4' & & \\
8 & 3' & & \\
\hline
74 & 8' & & \\
2 & 4' & 0'' & \\
\hline
77 & 0' & 0'' & \textit{Ans.}
\end{array}$$

3. How many solid feet in a stick of timber which is 25*ft.* 6*in.* long, 2*ft.* 7*in.* broad, and 3*ft.* 3 *inches* thick?

It is shown in § 85 that the number of solid or cubic feet in a solid, is equal to the length, breadth, and thickness multiplied together.

$$\begin{array}{rrrrl}
 & \textit{ft.} & & & \\
 & 25 & 6' & \text{length} & \\
 & 2 & 7' & \text{breadth} & \\
\hline
 & 51 & 0' & & \\
 & 14 & 10' & 6'' & \\
\hline
 & 65 & 10' & 6'' & \\
 & 3 & 3' & \text{thickness} & \\
\hline
 & 197 & 7' & 6'' & \\
 & 16 & 5' & 7'' & 6''' \\
\hline
\textit{Ans.} & 214 & 1' & 1'' & 6'''
\end{array}$$

4. Multiply 9*ft.* 2*in.* by 9*ft.* 6*in.*

Ans. 87*ft.* 1

5. Multiply 24*ft.* 10*in.* by 6*ft.* 8*in.*

Ans. 165*ft.* 6' 8".

6. Multiply 70*ft.* 9*in.* by 12*ft.* 3*in.*

Ans. 866*ft.* 8' 3".

7. How many cords and cord feet in a pile of wood 24 feet long, 4 feet wide, and 3*ft.* 6*in.* high?

Ans. 2 cords and 5 cord ft.

Note. It must be recollected that 16 solid feet make one cord foot § 84.

QUESTIONS.

§ 202. In Duodecimals what is the unit that is divided? How is it divided? How are these parts again divided? What are the parts called? How are duodecimals added and subtracted? How many of one denomination make 1 of the next greater?

§ 203. In multiplication how do you set down the multiplier? Where do you begin to multiply? How do you carry from one denomination to another?

§ 204. Repeat the four principles.

ALLEGATION MEDIAL.

§ 205. A merchant mixes 8*lb.* of tea worth 75*cts.* per pound, with 16*lb.* worth $1,02 per pound: what is the value of the mixture per pound?

The manner of finding the price of this mixture is called *Allegation Medial.* Hence,

ION MEDIAL teaches the method of *finding*
mixture when the simples of which it is
their prices, are known.

ple above, the simples 8lb. and 16lb

and also their prices per pound, 75*cts.* and $1,02, are known.

8*lb.* of tea at 75*cts.* per *lb.*	- -	6,00
16*lb.* - - $1,02 per *lb.*	- -	16,32
24 sum of simples	Total cost	$22,32

24)$22,32(93*cts.*
216
72
72
93*cts.*

Now if the entire cost of the mixture, which is $22,32, be divided by 24 the number of pounds, or sum of the simples, the quotient 93*cts.* will be the price per pound. Hence we have the following

RULE.

Divide the entire cost of the whole mixture by the sum of the simples: the quotient will be the price of the mixture.

Ex. 1. A farmer mixes 30 bushels of wheat worth 5*s* per bushel, with 72 bushels of rye at 3*s* per bushel, and with 60 bushels of barley worth 2*s* per bushel: what is the value of a bushel of the mixture?

30 bushels of wheat at 5*s* - -	150*s*
72 - - of rye at 3*s* - - -	216*s*
60 - - of barley at 2*s* - -	120*s*
162 - - - - - - - -	162)486(3*s*
	486

Ans. 3*s.*

2. A wine merchant mixes 15 gallons of wine at $1 per gallon, with 25 gallons of brandy worth 75 cts. per gallon: what is the value of a gallon of the compound?

Ans. 84cts.+

3. A grocer mixes 40 gallons of whiskey w

nothing: what is the value of a [illegible] ture?

[illegible]

4. A goldsmith melts together [illegible] carats fine, 6oz. of 20 carats fine, and [illegible] rats fine: what is the fineness of this [illegible]

Ans. [illegible]

5. On a certain day the mercury in th[illegible]eter was observed to average the followin[illegible] from 6 in the morning to 9, 64°; from 9 [illegible] from 12 to 3, 84°; and from 3 to 6, 70°: [illegible] the mean temperature of the day?

Ans. 73

ALLEGATION ALTERNATE.

§ 206. A farmer would mix oats worth 3s bushel with wheat worth 9s per bushel, so that mixture shall be worth 5s per bushel: what prop tion must be taken of each sort? The method of fi ing how much of each sort must be taken, is call *Allegation Alternate.* Hence,

ALLEGATION ALTERNATE *teaches the method of fi ing what proportion must be taken of several si ples, whose prices are known, to form a compound o given price.*

Allegation Alternate is the reverse of Allegati Medial and may be proved by it.

For the first example, let us take the one abo If oats worth 3s per bushel be mixed with wh worth 9s, how much must be taken of each sort th the compound may be worth 5s per bushel?

If th- [illegible] of the mixture sum of the [illegible]ples, it is

5 { 3 —— 4 Oats

9 —— 2 W

plain that it would be necessary to take just as much oats as wheat.

But since the price of the mixture is *nearer* to the price of the oats than to that of the wheat, less wheat will be required in the mixture than oats.

Having set down the prices of the simples under each other, and linked them together, we next set 5*s*, the price of the mixture, on the left. We then take the difference between 9 and 5 and place it opposite 3, the price of the oats, and also the difference between 5 and 3 and place it opposite 9, the price of the wheat. The difference standing opposite each kind shows how much of that kind is to be taken. In the present example, the mixture will consist of 4 bushels of oats and 2 of wheat; and any other quantities, bearing the same proportion to each other, as 8 and 4, 20 and 10, &c. will give a mixture of the same value.

PROOF BY ALLEGATION MEDIAL.

4 bushels of oats at 3*s* - -	12*s*
2 bushels of wheat at 9*s* - -	18*s*
6	6)30
Ans.	5*s.*

CASE I.

§ 207. To find the proportion in which several simples of given prices must be mixed together, that the compound may be worth a given price.

RULE.

I. *Set down the prices of the simples under each other, in the order of their values, beginning with the lowest.*

II. *Link the least price with the greatest, and th* *next least with the next greatest, and price of each simple which is less than t mixture is linked with one or more th and every one that is greater with one is less.*

III. *Write the difference between the mixture and that of each of the simples, price with which the particular simp then the difference standing opposite any or the sum of the differences when there is one, will express the quantity to be tak price.*

Ex. 1. A merchant would mix wines 18*s* and 22*s* per gallon, in such a way tha ture be worth 20*s* per gallon: how muc taken of each sort?

$$20.\left\{\begin{array}{ll}16 & 2\text{ at}\\ 18 & 2\text{ at}\\ 22 & 4+2\end{array}\right.$$

Ans. 2*gal.* at 16*s*, 2 at 18*s*, and 6 at any other quantities bearing the p of 2, 2 and 6.

2. What proportions of coffee 28*cts.* per pound, must be mix compound shall be worth

Ans. { In the proporti 20*cts.*, and 1

3. A goldsmi 24 carats fine: w that the mixture

Ans. 3 he

4. *What* 3,
old Madei
21s per

be mixed together, so that the mixture shall be worth 18*s* per gallon?

Ans. 6*gal*. at 10*s*, 3 at 14*s*, 4 at 21*s*,
and 8*gal*. at 24*s*.

CASE II.

§ 208. When a given quantity of one of the simples is to be taken.

RULE.

I. *Find the proportional quantities of the simples as in Case I.*

II. *Then say, as the number opposite the simple whose quantity is given, is to the given quantity, so is either proportional quantity, to the part of its simple to be taken.*

Ex. 1. How much wine at 5*s*, at 5*s* 6*d* and 6*s* per gallon, must be mixed with 4 gallons at 4*s* per gallon, so that the mixture shall be worth 5*s* 4*d* per gallon?

$$64\left\{\begin{array}{l}48\\60\\66\\72\end{array}\right. \qquad \left.\begin{array}{l}8 -\\2\\4\\16\end{array}\right\}$$

- simple whose quantity is known

proportional quantities.

Then 8 : 4 :: 2 : 1
8 : 4 :: 4 : 2
8 : 4 :: 16 : 8

Ans. 1*gal*. at 5*s*, 2 at 5*s* 6*d*, and 8 at 6*s*.

PROOF BY ALLEGATION MEDIAL.

4 *gal*. at 4*s*	per gal. -	192*d*
1 *gal*. at 5*s*	" -	60d
2 *gal*. at 5*s* 6*d*	" -	132d
8 *gal*. at 6*s*	" -	576d
15		15)960(64d price of

2. A farmer would mix 14 bushels of wheat, at $1,20 per bushel, with rye at 72cts., barley at 48cts., and oats at 36cts.: how much must be taken of each sort to make the mixture worth 64 cents per bushel?

Ans. 14*bu.* of wheat; 8*bu.* of rye; 4*bu.* of barley; and 28*bu.* of oats.

3. There is a mixture made of wheat at 4*s* per bushel, rye at 3*s*, barley at 2*s*, with 12 bushels of oats at 18*d* per bushel: how much is taken of each sort when the mixture is worth 3*s* 6*d*?

Ans. 96*bu.* of wheat; 12*bu.* of rye; 12*bu.* of barley; and 12 of oats.

4. A distiller would mix 40*gal.* of French brandy at 12*s* per gallon, with English at 7*s* and spirits at 4*s* per gallon: what quantity must be taken of each sort, that the mixture may be afforded at 8*s* per ga lon?

Ans. 40*gal.* French; 32 English; and 32 of Spirits.

CASE III.

§ 209. When the quantity of the compound given as well as the price.

RULE.

I. *Find the proportional quantities as in Case*

II. *Then say, as the sum of the proportional qu tities, is to the given quantity, so is each proportio quantity, to the part to be taken of each.*

Ex. 1. A grocer has four sorts of sugar wort 10*d*, 6*d* and 4*d* per pound: he would make a ture of 144*lb.* worth 8*d* per lb.: what quanti e *taken of* each sort?

8	4	4	12 : 144 : : 4 : 48
	6	2	12 : 144 : : 2 : 24
	10	2	12 : 144 : : 2 : 24
	12	4	12 : 144 : : 4 : 48
Sum of the proportional parts.		12	

Ans. 48*lb.* at 4*d*; 24*lb.* at 6*d*; 24*lb.* at 10*d*, and 48*lb.* at 12*d*.

PROOF BY ALLEGATION MEDIAL.

48*lb.* at 4 *d* - - -	192*d*
24*lb.* at 6 *d* - -	144*d*
24*lb.* at 10*d* - - -	240*d*
48*lb.* at 12*d* - - -	576*d*
144 - - -	144)1152(8*d*

The average cost is 8*d* as above.

2. A grocer having four sorts of tea worth 5*s*, 6*s*, 8*s* and 9*s* per pound, wishes a mixture of 87*lb.* worth 7*s* per pound: how much must be taken of each sort?

Ans. 14½*lb.* at 5*s*; 29*lb.* at 6*s*; 29*lb.* at 8*s* and 14½*lb.* at 9*s*.

3. A Vintner has four sorts of wine, viz. white wine at 4*s* per gallon, Flemish at 6*s* per gallon, Malaga at 8*s* per gallon, and Canary at 10*s* per gallon: he would make a mixture of 60 gallons to be worth 5*s* per gallon: what quantity must be taken of each?

Ans. 45*gal.* of white wine; 5*gal.* of Flemish; 5*gal.* of Malaga; and 5*gal.* of Canary.

4. A silversmith has four sorts of gold, viz. of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine: he would make a mixture of 42oz. of 17 carats fine: how much must be taken of each sort?

Ans. 4 of 24; 4 of 22; 4 of 20; and 30 of 15 carats fine.

QUESTIONS.

§ 205. What is Allegation Medial? How do you find the price of the mixture?

§ 206. What is Allegation Alternate? How do you prove Allegation Alternate?

§ 207. How do you find the proportional parts when the price only is given?

§ 208. What is the rule when a given quantity of one of the simples is to be taken?

§ 209. What is the rule when the quantity of the compound, as well as the price, is given?

INVOLUTION.

§ 210. If a number be multiplied by itself, the product is called the *second power*, or *square* of that number. Thus 4×4=16: the number 16 is the 2d power or square of 4.

If a number be multiplied by itself, and the product arising be again multiplied by the number, the second product is called the 3d *power*, or *cube* of the number. Thus 3×3×3=27: the number 27 is the 3d *power*, or *cube* of 3.

The term *power* designates the product arising from multiplying a number by itself a certain number of times, and the number multiplied is called the *root*.

Thus, in the first example above, 4 is the root, and 16 the square or 2d power of 4.

In the 2d example, 3 is the root, and 27 the 3d power or cube of 3. The first power of a number is the number itself.

§ 211. *Involution teaches the method of finding the powers of numbers.*

The number which designates the power to which *the root* is to be raised, is called the index or expo *nent* of the power. It is generally written on the right, and a little above the root. Thus, 4^2 expressed the second power of 4, or that 4 is to be multiplied by itself once: hence, $4^2=4\times4=16$.

For the same reason 3^3 denotes that 3 is to be raised to the 3d power, or cubed: hence

$3^3=3\times3\times3=27$: we may therefore write,

$4=4$ the 1st power of 4.
$4^2=4\times4=16$ the 2d power of 4.
$4^3=4\times4\times4=64$ the 3d power of 4.
$4^4=4\times4\times4\times4=256$ the 4th power of 4.
$4^5=4\times4\times4\times4\times4=1024$ the 5th power of 4.
&c. &c. &c.

§ 212. Hence, to raise a number to any power, we have the following

RULE.

Multiply the number continually by itself as many times less 1 *as there are units in the exponent: the last product will be the power sought.*

Ex. 1. What is the 3d power of 125?
$125\times125\times125=1953125$. *Ans.*

2. What is the cube of 7?
Ans. 343.

3. What is the square of 60?
Ans. 3600.

4. What is the 4th power of 5?
Ans. 625.

5. What is the 5th power of 9?
Ans. 59049.

6. What is the cube of 1?
Ans

7. What is the square of ¼?

Ans.

8. What is the cube of ,1?

Ans. ,0

9. What is the cube of ⅔?

Ans.

10. What is the square of ,01?

Ans. ,00

11. What is the square of 2,04?

Ans. 4,16

12. What is the 5th power of 10?

Ans. 1000

13. What is the cube of 2¼?

Ans. 11

QUESTIONS.

§ 210. If a number be multiplied by itself once, w the product called? If it be multiplied by itself twice, is the product called? What does the term power What is the root?

§ 211. What is Involution? What is the number which designates the power? Where is it written?

§ 212. How do you raise a number to any power?

EVOLUTION.

§ 213. We have seen § 211, that Involution te how to find the power when the root is given. lution is the reverse of Involution: it teaches to find the root when the power is known. root is that number which being multiplied by i a certain number of times, will produce the *power*.

e square root of a number is th

ber which being multiplied by itself once, will produce the given number.

The cube root of a number is that number which being multiplied by itself twice, will produce the given number. For example, 6 is the square of root of 36: because $6\times6=36$; and 3 is the cube root of 27, because $3\times3\times3=27$. The sign $\sqrt{\ }$ placed before a number denotes that its square root is to be extracted. Thus, $\sqrt{36}=6$. It is called the sign of the square root.

When we wish to express that the cube root is to be extracted, we place the figure 3 over the sign of the square root: thus, $\sqrt[3]{\ }$: $\sqrt[3]{27}=3$.

EXTRACTION OF THE SQUARE ROOT.

§ 215. To extract the square root of a number, is to find a number, which being multiplied by itself once will produce the given number. Thus $\sqrt{4}=2$; for $2\times2=4$. And $\sqrt{9}=3$; for $3\times3=9$.

Roots	1.	2,	3,	4,	5,	6,	7,	8,	9.
Squares	1	4	9	16	25	36	49	64	81.

From which we see that the square of either of the significant figures is less than 100, and hence the square root if any two figures will be less than 10. It is also evident that there are but nine perfect squares between 1 and 100.

CASE I.

§ 216. To extract the square root of a whole number.

RULE.

I. *Point off the given number into periods of two figures each, counted from the right, by setting a dot over the place of units, another over the place of hundreds and so on.*

II. *Find the greatest square in the first period on the left, and place its root on the right after the manner of a quotient in division. Subtract the square of the root from the first period, and to the remainder bring down the second period for a dividend.*

III. *Double the root already found and place it on the left for a divisor. Seek how many times the divisor is contained in the dividend, exclusive of the right hand figure, and place the figure in the root and also at the right of the divisor.*

IV. *Multiply the divisor, thus augmented, by the last figure of the root, and subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

V. *Double the whole root already found, for a new divisor, and continue the operation as before, until all the periods are brought down.*

EXAMPLES.

1. What is the square root of 263169 ?

We first place a dot over the 9, making the right hand period 69. We then put a dot over the 1 and also over the 6, making three periods.

```
      26̇31̇69̇(513
      25
  101)131
      101
 1023)3069
      3069
```

The greatest perfect square in 26, is 25, the root of which is 5. Placing 5 in the root, subtracting its square from 26, and bringing down the next period *31, we have* 131 for a dividend, and by doubling the

e *have* 10 for a divisor. Now 10 is contain-

3, 1 time. Place 1 both in the root and in the divisor: then multiply 101 by 1; subtract the product and bring down the next period.

We must now double the whole root 51 for a new divisor, or we may take the first divisor after having doubled the last figure 1; then by dividing we obtain 3, the third figure of the root.

Note 1. There will be as many figures in the root as there are periods in the given number.

Note 2. If the given number has not an exact root, there will be a remainder after all the periods are brought down, in which case ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

2. What is the square root of 36729 ?

In this example there are two periods of decimals, which give two places of decimals in the root.

```
         36729(191,64+.
         1
       29)267
          261
        381)629
            381
       3826)24800
             22956
      38324)184400
             153296
              31104 Rem.
```

3. What is the square root of 106929 ?

Ans. 327.

4. What is the square root of 2268741 ?

Ans. 1506,23+.

5. What is the square root of 7596796 ?

Ans. 2756,22+.

6. What is the square root of 36372961 ?

Ans. 6

7. What is the square root of 2207

CASE II.

§ 217. To extract the square root of a fraction.

RULE.

I. *Annex one cipher, if necessary, so that the number of decimal places shall be even.*

II. *Point off the decimal into periods of two figures each, by putting a point over the place of tenths, a second over the place of thousandths, &c.: then extract the root as in whole numbers, recollecting that the number of decimal places in the root will be equal to the number of periods in the given decimal.*

Ex. 1. What is the square root of ,5 ?

```
         ,50(,707+
          49
     140)100
          000
    1407)10000
          9849
           151 Rem.
```

Note. When there is a decimal and a whole number joined together the same rule will apply.

2. What is the square root of 3271,4207 ?
Ans. 57,19+.

3. What is the square root of 4795,25731 ?
Ans. 69,247+.

4. What is the square root of 4,372594 ?
Ans. 2,091+.

What is the square root of ,00032754 ?
Ans. ,01809+.

CASE III.

§ 218. To extract the square root of a vulgar fraction.

RULE.

I. *Reduce mixed numbers to improper fractions and compound fractions to simple ones, and then reduce the fraction to its lowest terms.*

II. *Extract the square root of the numerator and denominator separately, if they have exact roots; but when they have not, reduce the fraction to a decimal and extract the root as in Case II.*

1. What is the square root of $\frac{2304}{5184}$? *Ans.* $\frac{2}{3}$.

2. What is the square root of $\frac{2704}{4225}$? *Ans.* $\frac{4}{5}$.

3. What is the square root of $\frac{9216}{12544}$? *Ans.* $\frac{6}{7}$.

4. What is the square root of $\frac{275}{341}$? *Ans.* ,89802+.

5. What is the square root of $\frac{357}{476}$? *Ans.* 86602+.

6. What is the square root of $\frac{478}{549}$? *Ans.* ,93309+

EXTRACTION OF THE CUBE ROOT.

§ 219. To extract the cube root of a number is to find a *second* number which being multiplied into itself twice, shall produce the given number.

Thus, 2 is the cube root of 8; for 2×2×2=8: and 3 is the cube root of 27; for 3×3×3=27.

Roots	1	2,	3,	4,	5,	6,	7,	8,	9.
Cubes	1	8	27	64	125	216	343	512	729

CASE I.

§ 220. To extract the cub[illegible]oot of [illegible]ber.

RULE.

I. *Point off the given number into pe[illegible] figures each, by placing a dot over the p[illegible] a second over the place of thousands, an[illegible] left: the left hand period will often contain [illegible] three places of figures.*

II. *Seek the greatest cube in the first period, and set its root on the right, after the manner of a quotient in division. Subtract the cube of this figure of the root from the first period, and to the remainder bring down the first figure of the next period, and call the number the dividend.*

III. *Take three times the square of the root just found for a divisor and see how often it is contained in the dividend and place the quotient for a second figure of the root. Then cube the figures of the root thus found, and if their cube be greater than the two first periods of the given number, diminish the last figure, but if it be less subtract it from the two first periods, and to the remainder bring down the first figure of the next period, for a new dividend.*

IV. *Take three times the square of the whole root for a new divisor, and seek how often it is contained in the new dividend: the quotient will be the third figure of the root. Cube the whole root and subtract the result from the first three periods of the given number, and proceed in a similar way for all the periods.*

Ex. 1. What is the cube root of 99252847?

$\dot{9}925\dot{2}84\dot{7}(463$

$4^3 = 64$

$4^2 \times 3 = 48)\overline{352}$ dividend

Two first periods - $\dot{9}925\dot{2}$

$(46)^3 = 46 \times 46 \times 46 =$ 97336

$3 \times (46)^2 = 6348$) $\overline{19168}$ 2d dividend

The first three periods $\dot{9}925\dot{2}84\dot{7}$.

$(463)^3$ - $= 99252847$.

Ans. 463.

2. What is the cube root of 389017? *Ans.* 73.

3. What is the cube root of 5735339? *Ans.* 179.

4. What is the cube root of 32461759? *Ans.* 319.

5. What is the cube root of 84604519? *Ans.* 439.

6. What is the cube root of 259694072? *Ans.* 638.

7. What is the cube root of 48228544? *Ans.* 364.

8. What is the cube root of 27054036008? *Ans.* 3002.

CASE II.

§ 221. To extract the cube root of a decimal fraction.

RULE.

Annex ciphers to the decimal, if necessary, so that it shall consist of 3, 6, 9, &c. places. Then place the first point over the place of tenths, the second over the place of ten thousandths, and so on over ever *third place to the right; after which [illegible] as in whole numbers.*

Note 1. There will be as many *decimal* [illegible] the root as there are periods in *the given* [illegible]

Note 2. The same rule applies *when* [illegible] number is composed of a whole *number* [illegible] *and* [illegible]cimal.

Note 3. If in extracting the root of *a* [illegible] there is a remainder, after all the periods have [illegible] brought down, periods of ciphers may be ann[illegible] by considering them as decimals.

EXAMPLES.

1. What is the cube root of ,157464?

Ans. ,54.

2. What is the cube root of ,870983875?

Ans. ,955.

3. What is the cube root of 12,977875?

Ans. 2,35.

CASE III.

§ 222. To extract the cube root of a vulgar fraction.

RULE.

I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms.*

II. *Then extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal, and extract the root as in* [illegible] *Case.*

EXAMPLES.

[illegible]at is the cube root of [illegible]?

Ans. [illegible]

2. What is the cube root of $12\frac{19}{27}$?

Ans. $2\frac{1}{3}$.

3. What is the cube root of $31\frac{15}{343}$?

Ans. $3\frac{1}{7}$.

4. What is the cube root of $\frac{324}{1500}$?

Ans. $\frac{3}{5}$.

5. What is the cube root of $\frac{4}{7}$?

Ans. ,829+.

6. What is the cube root of $\frac{5}{9}$?

Ans. 822+.

7. What is the cube root of $\frac{2}{3}$?

Ans. ,873+.

QUESTIONS.

§ 213. What is Evolution? What does it teach?

§ 214. What is the square root of a number? What is the cube root of a number? Make the sign denoting the square root? How do you denote the cube root?

§ 215. What is required when we wish to extract the square root of a number? What number is greater than the square of either of the significant figures?

§ 216. How do you extract the square root of a whole number? How many figures will there be in the root? If the given number has not an exact root, what may be done?

§ 217. How do you extract the square root of a decimal fraction? When there is a decimal and whole number joined together?

§ 218. How do you extract the square root of a vulgar fraction?

§ 219. What is required when we are to extract the cube root of a number?

§ 220. How do you extract the cube root of a whole number?

§ 221. How do you extract the cube root of a decimal fraction? How many decimal places will there be in the root? Will the same rule apply when there is a whole number and a decimal? In extracting the root if there be a remainder, what *may* be done?

*§ 222. How do you extract the cube root of a vulgar fra
tion?*

ARITHMETICAL PROGRESSION.

§ 223. If we take any number, as 2, we can, by the continued addition of any other number, as 3, form a *series* of numbers; thus,

2, 5, 8, 11, 14, 17, 20, 23, &c. in which each number is formed by the addition of 3 to the preceding number.

This series of numbers may also be formed by subtracting 3 continually from the largest number; thus,

23, 20, 17, 14, 11, 8, 5, 2.

A series of numbers formed in either way is called an *Arithmetical Series*, or an *Arithmetical Progression;* and the number which is added or subtracted is called the common difference.

When the proportion is formed by the continued addition of the common difference, it is called an *ascending* series; and when it is formed by the subtraction of the common difference, it is called a *descending* series; thus,

2, 5, 8, 11, 14, 17, 20, 23, is an ascending series.
23, 20, 17, 14, 11, 8, 5, 2. is a descending series.

The several numbers are called the terms of the progression: the first and last terms are called the *extremes*, and the intermediate terms are called the *means.*

§ 224. In every arithmetical progression there are five things which are considered, any three of which being given or known, the remaining two can be determined.

They are

1st the first term,
2d the last term,
3d the common difference,
4th the number of terms,
5th the sum of all the terms.

By considering the manner in which the ascending progression is formed, we see that the 2d term is obtained by adding the common difference to the first term; the 3d by adding the common difference to the 2d; the 4th by adding the common difference to the 3d, and so on; *the number of additions being* 1 *less than the number of the term found.*

But instead of making the additions, we may multiply the common difference by the number of additions, that is by 1 less than the number of terms, and add the first term to the product. Hence we have

CASE I.

§ 225. Having given the first term, the common difference, and the number of terms, to find the last term.

RULE.

Multiply the common difference by 1 *less than the number of terms, and to the product add the first term.*

Ex. 1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

18 number of terms less 1.
2 common difference
36
3 1st term
39 last term.

Ans. 39.

2. A man bought 50 yards of cloth for which he was to pay 6 cents for the first yard, 9 cents for the 2d, 12 cents for the 3d, and so on increasing by the common difference 3: how much did he pay for the last yard?

Ans. $1,53.

3. A man puts out $100 at simple interest, at 7 per cent.; at the end of the first year it will have increased to $107, at the end of the 2d year to $114, and so on, increasing $7 each year: what will be the amount at the end of 16 years?

Ans. $205.

Since the last term of an arithmetical progression is equal to the first term added to the product of the common difference by 1 less than the number of terms, it follows, that the difference of the extremes will be equal to this product, and that the common difference will be equal to this product divided by 1 less than the number of terms. Hence we have

CASE II.

¶ 228. Having given the two extremes and the number of terms of an arithmetical progression, to find the common difference.

RULE.

Subtract the less extreme from the greater and divide the remainder by 1 less than the number of terms, the quotient will be the common difference.

Ex. 1. The extremes are 4 and 104, and the number of terms 26: what is the common difference?

$$\begin{array}{r} 104 \\ \underline{4} \\ 26-1=25)\overline{100}(4 \\ \underline{100} \end{array}$$

Ans. 4.

2. A man has 8 sons, the youngest is 4 years old and the eldest 32, their ages increase in arithmetical progression: what is the common difference of their ages?

$32-4=28$: then $8-1=7)28(4$

Ans. 4.

3. A man is to travel from New York to a certain place in 12 days, and to go 3 miles the first day, increasing every day by the same number of miles, so that the last day's journey may be 58 miles: required the daily increase.

Ans. 5 miles.

If we take any arithmetical series, as

3 5 7 9 11 13 15 17 19, &c.
19 17 15 13 11 9 7 5 3 by reversing the order
22 22 22 22 22 22 22 22 22 of the terms.

Here we see that the sum of the terms of these two series is equal to 22, the sum of the extremes, multiplied by the number of terms; and consequently, the sum of either series is equal to the sum of the two extremes multiplied by half the number of terms. Hence we have

CASE III.

§ 227. To find the sum of all the terms of an arithmetical progression.

RULE.

Add the two extremes together and multiply their sum by half the number of terms, the product will be the sum of the series.

Ex. 1. The extremes are 2 and 100, and the number of terms 22: what is the sum of the series?

2 1st term
100 last term
102 sum of extremes
11 half the number of terms
1122 sum of series.

Ans. 1122.

2. How many strokes does the hammer of a clock strike in 12 hours?

Ans. 78.

3. If 100 eggs were placed in a right line, exactly one yard from each other, and the first, one yard from a basket: what distance will a man travel who gathers them up singly, and places them in the basket?

Ans. 5 miles, 1300 yards.

GEOMETRICAL PROGRESSION.

§ 228. If we take any number, as 3, and multiply it continually by any other number, as 2, we form a series of numbers, thus,

3 6 12 24 48 96 192, &c. in which each number is formed by multiplying the number before it by 2.

This series may also be formed by dividing continually the largest number 192 by 2. Thus,

192 96 48 24 12 6 3

A series formed in either way is called a Geometrical Series, or a Geometrical Progression, and the number by which we continually multiply or divide, is called the *common ratio.*

When the series is formed by multiplying continually by the common ratio, it is called an *ascending series;* and when it is formed by dividing continually by the common ratio, it is called a *descending series.*

Thus,

3 6 12 24 48 96 192 is an ascending series.
192 96 48 24 12 6 3 is a descending series.

The several numbers are called the *terms* of the progression.

The first and last terms are called the *extremes*, and the intermediate terms are called the *means*.

§ 229. In every Geometrical, as well as in every Arithmetical Progression, there are five things which are considered, any three of which being given or known, the remaining two can be determined.

They are,

1st the first term,
2d the last term,
3d the common difference,
4th the number of terms,
5th the sum of all the terms.

By considering the manner in which the ascending progression is formed, we see that the second term is obtained by multiplying the first term by the common ratio; the 3d term by multiplying this product by the common ratio, and so on, the number of multiplications being one less than the number of terms. Thus,

$3=1$ 1st term,
$3\times2=6$ 2d term,
$3\times2\times2=12$ 3d term,
$3\times2\times2\times2=24$ 4th term, &c. for the other terms.

But $2\times2=2^2$, $2\times2\times2=2^3$, and $2\times2\times2\times2=2^4$.

Therefore, any term of a progression is equal to the first term multiplied by the ratio raised to a power 1 less than the number of the term.

CASE I.

§ 230. *Having given the first term, the common ratio, and the number of terms, to find the last ter*

RULE.

Raise the ratio to a power whose exponent is 1 less than the number of terms, and then multiply the power by the first term, the product will be the last term.

Ex. 1. The first term is 3 and the ratio 2; what is the 6th term?

$$2\times2\times2\times2\times2=2^5=64$$

3 1st term

Ans. 192

2. A man purchased 12 pears: he was to pay 1 farthing for the first, 2 farthings for the 2d, 4 for the 3d, and so on, doubling each time: what did he pay for the last?

Ans. £2 2*s* 8*d*.

3. A gentleman dying left nine sons, and bequeathed his estate in the following manner: to his executors £50; his youngest son to have twice as much as the executors, and each son to have double the amount of the son next younger: what was the eldest son's portion?

Ans. £25600.

CASE II.

§ 231. Having given the two extremes to find the sum of the series.

RULE.

Subtract the less extreme from the greater, divide the remainder by 1 less than the ratio, and to the quotient add the greater extreme: the sum will be the sum of the series.

1. The first term is 3, the ratio 2, and the last 92: what is the sum of the series?

192—3=189 difference of the extremes,

1=1)189(189; then 189+192=381 Ans.

2. A gentleman married his daughter on New Year's day, and gave her husband 1*s* towards her portion, and was to double it on the first day of every month during the year: what was her portion?

Ans. £204 15*s*.

QUESTIONS.

§ 223. How do you form an Arithmetical Series? What is the common difference? What is an ascending series? What a descending series? What are the several numbers called? What are the first and last terms called? What are the intermediate terms called?

§ 224. In every Arithmetical Progression how many things are considered? What are they?

§ 225. How do you find the last term when the first term and common difference are known?

§ 226. How do you find the common difference, when you know the two extremes and number of terms?

§ 227. How do you find the sum of the series?

§ 228. How do you form a Geometrical Progression? What is the common ratio? What is an ascending series?—What a descending series? What are the several numbers called? What are the first and last terms called? What are the intertermediate terms called?

§ 229. How many things are considered in a Geometrical Progression? What are they?

§ 230. How do you find the last term when the first term and common difference are known?

§ 231. How do you find the sum of the series?

PROMISCUOUS QUESTIONS.

1. A merchant bought 13 packages of goods, for which he paid $326: what will 39 packages cost at the same rate?

Ans. $978.

2. Two merchants, A and B traded together; A put in £320 for 5 months, and B £460 for 3 months

they gained £100: how much should each one receive?

Ans. A £53 13*s* 9[illegible]*d*, B £46 6*s* 2[illegible]*d*.

3. If I buy 1000 Ells Flemish of linen, for £90, what must it be sold for per Ell English, to make £10 by the purchase? *Ans.* 3*s* 4*d*.

4. What number taken from the square of 54 will leave 19 times 46? *Ans.* 2042.

5. If $\frac{3}{4}$ of a gallon of wine cost $\frac{3}{8}$ of a £, what will $\frac{4}{5}$ of a tun cost?

Ans. £105.

6. If an officer's salary be £48 per annum, how much will he receive in 232 days?

Ans. £30 10*s* 2$\frac{1}{4}$*d*+.

7. If a gentleman spends, one day with another, £1 7*s* 10$\frac{1}{2}$*d*, and at the end of the year has saved £340, what is his yearly income?

Ans. £848 14*s* 4$\frac{1}{2}$*d*.

8. If 8 cannons expend, in one day, 48 barrels of powder, how much will 24 cannons expend in 22 days?

Ans. 3168*bar.*

9. What number is that which being multiplied by $\frac{2}{3}$ will produce $\frac{1}{4}$?

Ans. $\frac{3}{8}$.

10. A person dying divided his property between his widow and his four sons: to his widow he gave $1780, and to each of his sons $1250: he had been 25$\frac{1}{2}$ years in business, and had cleared on an average $126 a year: how much had he when he began busin— ? *Ans.* $3567.

esieged garrison consisting of 360 men
oned for 6 months, but hearing of no re
of 5 months, dismissed so many of th

ison that the remaining provision lasted 5 months: many men were sent away?

Ans. 288.

2. What number added to 11$\frac{5}{7}$, will produce $\frac{37}{56}$?

Ans. 24$\frac{53}{56}$.

3. What number multiplied by $\frac{3}{7}$, will produce ?

Ans. 26$\frac{46}{51}$.

4. A man had 12 sons, the youngest was 3 years and the eldest 58, and their ages increased in metical progression: what was the common difference of their ages?

Ans. 5 years.

5. A snail in getting up a pole, 20 feet high, was rved to climb up 8 feet every day, but to descend et every night: in what time did he reach the of the pole?

Ans. 4 days.

6. Two persons, A and B are indebted to C; A s $2173, which is the least debt, and the difference of the debts is $371: what is B's debt?

Ans. $2544.

7. What is the difference between twice four and and twice forty-four: also between twice five fifty and twice fifty-five?

Ans. 40 and 50.

8. A lady being asked her age, and not wishing ive a direct answer, said, I have 9 children, and e years elapsed between the birth of each of n; the eldest was born when I was 19 years old, the youngest is now exactly 19: what was her ?

Ans. 62 years.

9. What number added to the 43d part of 4429, make the sum 240?

Ans. 137.

20. A man went to sea at 17 years of age; 8 years after he had a son born, who lived 46 years and died before his father: after which the father lived twice twenty years and died: what was the age of the father?

Ans. 111 years.

21. A brigade of horse consisting of 384 men, is to be formed into a solid body consisting of 32 men in front; how many ranks will there be?

Ans. 12.

22. A room 30 feet long and 18 feet wide is to be covered with painted cloth $\frac{3}{4}$ of a yard in width: how many yards will cover it?

Ans. 80.

23. A, B and C trade together and gain $120, which is to be shared according to each one's stock; A put in $140, B $300, and C $160: what is each man's share?

Ans. A's $28, B's $60, and C's $32.

24. There is a stone which measures 4 feet 6 inches long, 2 feet 9 inches broad, and 5 feet 4 inches deep: how many solid feet does it contain?

Ans. 66 feet.

25. Two men depart from the same place and travel in different directions; one goes 7 miles and the other 11 miles per day: how far will they be apart at the end of the 12th day?

Ans. 216 miles.

26. How many planks 15 feet long and 15 inches wide, will floor a barn $60\frac{1}{2}$ feet long and $33\frac{1}{2}$ feet wide?

Ans. $108\frac{1}{11}$.

man owned $\frac{3}{5}$ of a mine, and sold $\frac{3}{4}$ of his
$1710: what was the value of the

Ans. $3800.

28. The swiftest velocity of a cannon ball, is about 2000 feet in a second of time. In what time, at that rate, would it be in moving from the earth to the sun, admitting the distance to be 95 millions of miles, and the year to contain 365 days 6 hours.

Ans. $7\frac{12457}{13149}$ years.

29. The slow or parade step is 70 paces per minute, at 28 inches each pace: how fast is that per hour?

Ans. $1\frac{113}{132}$ miles.

30. A wall of 700 yards in length was to be built in 29 days. Twelve men were employed on it for 11 days, and only completed 220 yards. How many men must be added to complete the wall in the required time?

Ans. 4.

31. How far will 500 millions of guineas reach, when laid down in a straight line touching one another, supposing each guinea to be an inch in diameter?

Ans. 7891*mi.* 728*yd.* 2*ft.* 8*in.*

32. A gentleman whose annual income is £1500, spends 20 guineas a week: does he save or run in debt, and how much?

Ans. He saves £408 per annum.

33. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence: did he make or lose, and how much?

Ans. He lost 6 pence.

34. My factor sends me word he has bought goods to the value of £500 13*s* 6*d* upon my account; what will his commission come to at $3\frac{1}{2}$ per cent.?

Ans. £17 10s $5\frac{1}{2}$d +

12 weeks, at the rate of 20 ounces a day for each man; how many men will the same provisions maintain for 20 weeks, at the rate of 8 ounces a day for each man?

Ans. 2250 men.

36. A younger brother received $8400, which was just $\frac{7}{9}$ of his elder brother's fortune: what was the father worth?

Ans. $19200.

37. If 20 men can perform a piece of work in 12 days, how many men will accomplish three times as much in one-fifth of the time?

Ans. 300.

38. Suppose that I have $\frac{3}{16}$ of a ship worth $1200; what part have I left after selling $\frac{2}{5}$ of $\frac{4}{9}$ of my share, and what is it worth?

Ans. $\frac{37}{240}$ left, worth $986,66+

39. What number is that which being multiplied by $\frac{2}{3}$ of $\frac{3}{4}$ of $1\frac{1}{2}$, the product will be 1?

Ans. $1\frac{1}{3}$.

40. What number is that which being multiplied by three thousandths, the product will be 2637?

Ans. 879000.

41. What length must be cut off a board $8\frac{1}{2}$ inches broad to contain a square foot, or as much as 12 inches in length and 12 in breadth?

Ans. $16\frac{16}{17}$ inches.

42. A man exchanged 70 bushels of rye, at $0,92 per bushel, for 40 bushels of wheat, at $1,37½ per bushel, and received the balance in oats, at $0,40 per bushel: how many bushels of oats did he receive?

Ans. 23½.

4 orse and saddle together are worth $132,

and the horse is worth 10 times as much as the saddle: what is the value of the horse?

Ans. $120.

44. Four persons trade together on a capital of $6000, of which A put in ½, B ¼, C ⅙ and D the rest; at the end of 4 years they had gained $4728: what was each one's share of the gain?

Ans. {
A's $2364
B's $1182
C's $ 788
D's $ 394

45. A farmer being asked how many sheep he had, answered, that he had them in five fields, in the first he had ¼ of his flock, in the second ⅙, in the 3d ⅛, in the 4th 1/12, and in the 5th 450: how many had he?

Ans. 1200.

46. The circumference of the earth is 360 degrees, and each degree is 69½ miles, how long would a man be in travelling round it, who travelled at the rate of 20 miles a day, the year being reckoned at 365 days 6 hours?

Ans. 3 years, 155¼ days.

47. How many bricks 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet; also a yard that is 50 feet square?

Ans. 22500—2d yard 11250.

48. Sound travels about 1142 feet in a second. Now if the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?

Ans. 9mi. 5fur. 34rd.+

49. Two persons depart from the same place, one travels 32, and the other 36 miles a day: if they travel in the same direction, how far will they

apart at the end of 19 days, and how far if they travel in contrary directions?

Ans. { 76 miles same direction.
{ 1292 miles opposite directions.

50. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of the trees bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears: how many trees are there in the orchard?

Ans. 2400.

APPENDIX.

RELATING TO BUSINESS.

FORMS OF ORDERS.

Messrs. M. James & Co.

Please pay John Thompson, or order, five hundred dollars, and place the same to my account.

Peter Worthy.

New York, June 1, 1833.

Mr. Joseph Rich,

Please pay the bearer sixty-one dollars and twenty cents, in goods from your store, and charge the same to the account of your

Obedient Servant,

John Parsons.

New York, July 1, 1833.

* * *

FORMS OF RECEIPTS.

Receipt for Money on Account.

eived, New York, June 2d, 1833, of John Ward, sixty

on account.

J. P. Fa

Receipt for Money on a Note.

Received New York, June 5, 1833, of Leonard Walsh, hundred and forty dollars, on his note for one thousand llars, dated New York, January 1, 1833.

640,00. J. N. Weeks.

* * *

FORMS OF NOTES.

Negotiable Note.

o. 1.
5,50. New York, May 1, 1833.

For value received I promise to pay, on demand, Abel Bond, or order, twenty-five dollars and fifty cents.

Reuben Holmes.

Note Payable to Bearer.

o. 2.
75,39. New York, May 2, 1833.

For value received, I promise to pay, six months ter date, to John Johns, or bearer, eight hundred and sevty-five dollars and thirty-nine cents.

Pierce Penny.

Note by two Persons.

o. 3.
59,27. New York, June 2, 1833.

For value received, we, jointly and severally, omise to pay to Richard Ricks, or order, on demand, six ndred and fifty-nine dollars and twenty-seven cents.

Enos Allan,
John Allan.

Note Payable at a Bank.

No. 4.
20,25.

Sixty days after date, I promise to pay John Anrson, or order, at the Bank of the United States, twenty llars and twenty-five cents, for value received.

Jesse Stokes.

Remarks relating [illegible]

1. The person who signs a note, is called the drawer or maker of the note: thus, Reuben Holmes is the drawer of note No. 1.

2. The person who has the rightful possession of a note, is called the holder of the note.

3. A note is said to be *negotiable* when it is made payable to A B, or order, (See No. 1.) Now if Abel Bond to whom this *note* is made payable, writes his name on the back of it, he is said to *endorse* the note, and he is called the endorser; and when the note becomes due, the holder must first demand payment of the maker, Reuben Holmes, and if he declines paying it, the holder may then require payment of Abel Bond, the endorser.

4. If the note is made payable to A B, or bearer, then the drawer alone is responsible, and he must pay to any person who holds the note.

5. The time at which a note is to be paid should always be named, but if no time is specified, the drawer must pay when required to do so, and the note will draw interest after the payment is demanded.

6. When a note, payable at a future day, becomes due, it will draw interest, though no mention is made of interest.

7. In each of the States there is a *rate* of interest established by law, which is called the legal interest, and when no rate is specified, the note will always draw legal interest. If a rate *higher* than legal interest be taken, the drawer, in most of the States, is not bound to pay the note.

8. If two persons jointly and severally give their note, (See No. 3) it may be collected of either of them.

9. The words "For value received," should be expressed in every note.

10. When a note is given, payable on a fixed day, and in a specific article, as in wheat or rye, payment must be offered at the specified time, and if it is not, the holder can demand the value in money.

A PRACTICAL SYSTEM

OF

BOOK KEEPING.

PERSONS transacting business find it necessary to write down the articles bought or sold, together with their prices and the names of the persons with whom the bargains are made.

BOOK KEEPING is the method of recording such transactions in a regular manner. It is divided into two kinds, called Single Entry and Double Entry. The method by Single Entry is the most simple, and answers for all common business. This method we will explain.

Book Keeping by Single Entry requires two books, a Day Book and a Leger; and when cash sales are extensive, an additional book is necessary, which is called a Cash Book.

DAY BOOK.

This book should contain a full history of the business transactions, in the precise order in which they may have occurred.

The transfer of an account from the Day Book to the Leger, is called *posting* the account.

Each page of the Day Book should be ruled with two columns on the right hand of the page, one for dollars, and one for cents, and one column on the left hand for entering the page of the Leger on which the account may be posted.

The Day Book should begin with the name of the owner, and place of residence; and then should follow a full account of the transactions in business in the exact order in which they may have occurred.

The name of the person, or customer, is first written with the term *Dr.* or *Cr.* opposite, according as he becomes a debtor or creditor by the transaction.

Generally, the person who receives is Debtor, and the per*son who parts* with his property is the Creditor.

debtor to the amount of the goods, a[illegible] goods should specified particularly in making the charge.

If I buy goods, on credit of C D, I enter C D Cr. to amount of the goods, taking care to specify the goods in charge.

If I pay money for, or on account of another person, h becomes Dr. to me for the amount paid.

If a person pays money for me, he becomes Cr. to me fo the amount paid.

The Day Book and Leger are generally designated, Da Book A, Day Book B, Leger A, Leger B, &c.: for whe one book, in the course of business, is filled with charges, new one is taken.

DAY BOOK A.

Page 1. *Edward P. Nixon, New York, June* 1, 1833.

Folio Leger.	*New York, June* 1*st*, 1833.	$	ct
	George Wilson, - - - - Dr.		
× 1.	To 11 cwt. of sugar at $9 per cwt. $99,00 To 66lb. of coffee at 20 cts. pr. lb. 13,20	112	2
	Henry Jones, - - - - - Dr.		
× 1.	To balance of former account, $159,10 To 5 gals. of molasses at 32 cts. per gal. - - - - - - - - 1,60	160	7
	2*d*.		
	Charles Patch, - - - - - Dr.		
× 1.	To Cash, - - - - - - - - $327,09 To one hogshead of gin, - - 124,02	451	1
	John Dodson, - - - - - - Dr.		
× 1.	To 10 pieces of cloth at $4,50 per piece. - - - - - - - - $45,00 To 15 yards of calico at 25 cts. per yard. - - - - - - - - 3,75	48	7
	3*d*.		
	William Wells, - - - - - Dr.		
× 1.	To 400lb. of beef at $3,25 per 100lb. - - - - - - - - - $ 33,00 *To* 6000lb. of cheese at $10 cwt. 600,00	[illegible]	
	Henry Jones, - - - - - - Cr.		
	By Cash, - - - - - - - - - - - - -		

Page 2. [illegible]ork, *June 5th*, 1833.

Folio Leger.			$	cts
	George Wilson, - - - - - Cr.			
×1.	By Cash, - - - - - - - - -	$100		
	By his note of this date for -	12,20	112	20
	William Wells, - - - - - Cr.			
×1.	By Cash, - - - - - - - - - - - -		633	00
	6th.			
	John McNeil, - - - - - - Dr.			
×1.	To Cash, - - - - - - - - - -	$275,10		
	To one horse, - - - - - -	125,00		
	To 85lb. of butter at 20 cts. per lb.	17,00	417	10
	Daniel Judson, - - - - - Dr.			
×1.	To 3 *hhd.* of molasses at $20 each,	$60,00		
	To 3 *bar.* of salted shad at $8 per barrel. - - - - - - - - - -	24,00		
	To 4 kegs of raisins at $2 per kg.	8,00	92	00
×1.	Charles Patch, - - - - - Cr.			
	By Cash, - - - - - - - - -	$400,00		
	By his note of this date, due Aug. 1, 1833, - - - - - -	51,11	451	11
	8th.			
	Daniel Judson, - - - - - Cr.			
×1.	By 116 *lb.* of beef at 8 cts. per *lb.*	$ 9,28		
	By 50 bu. of oats at 37 cts per bush.	18,50	27	78
	Jared Newton, - - - - - Dr.			
×1.	To 1 piece of linen 36 yards, -	$42,50		
	To 3*yd.* of broadcloth at 4,50 per *yd*	13,50		
	To 46 *lb.* of nails at 6 cts. - - -	2,76	58	76
	10th.			
	Jared Newton, - - - - - Cr.			
×1.	By Cash, - - - - - - - - -	$37,50		
	By do. - - - - - - - - - - -	21,26	58	76
	John Dodson, - - - - - - Cr.			
	By Cash, - - - - - - - - -	$24,50		
×1.	By 20 *lb.* of butter at 18 cts. per *lb.*	3,60		
	By his note of this date, on demand,	20,65	48	75
	John McNeill, - - - - - Cr.			
×1.	By Cash, - - - - - - - - - - - - -		100	0

The Leger is a book into which is co
form, all the scattered accounts from the
Two columns should be ruled on the rig
the Leger, one for dollars and one for c
also be two columns on the left, one to ins
transaction, and the other for inserting the pag
Book from which the account is transferred.

Two pages of the Leger, facing each other, a
used in stating an account, though sometimes a p
ded into two equal parts, in which case each p
considered as forming a separate page. The name
son with whom the account is stated should be
large letters at the top of the left hand page.

The Debits are entered on the left hand pag
Credits on the other page directly opposite. Th
between the debits and credits, is always entered
least sum, when the account is closed, and is call
ance, as in the account of John McNeill.

At the top of the left hand column, we enter
under which, we enter the day of the month on
transaction took place; and in the adjoining colu
ter the page of the Day Book from which the
transferred.

When there are several articles charged in the
we need not specify them all, but may enter them
ger under the general name of "Sundries." Havi
the account, we enter the page of the Leger to wh
been transferred, in the left hand column of the
and opposite the account, and make a mark × to
eye and show that the account is posted.

We begin posting with the account of Georg
who stands charged on the Day Book with $112,
posted the first charge, we run the eye carefully th
Day Book, to see if there are any other charges ag
and find that in page 2 he is credited by 100 dollar
a note for $12,20. These items we enter in the
the credit side, and as the debits and credits
account is balanced. We proceed in the same

all the accounts entered in the Day Book. No erasures should ever be made in the account books. When an error is discovered, if it be in favor of the customer he should be charged with the amount, and if against him, he should be credited with the amount. In posting the account of Jared Newton, a mistake was made against him of $21,26, which was rectified by crediting him with the amount.

When a charge is entered on the wrong side of the book, as when a person is charged with that for which he ought to have been credited, *twice* the amount must be entered on the other side of the book to make the account right.

Every Leger should have an Index, where the names of all persons, who have accounts in the Leger, should be arranged in alphabetical order.

When a Leger is filled, all the accounts are balanced, and when we transfer the balances to a new Leger, we charge "To balance from Leger A, page" —.

INDEX TO LEGER.

A.	*Folio*	B.	*Folio*	C.	*Folio*
D.		M.		P.	
Dodson, John	1	McNeill, John	1	Patch, Chas.	1
J.		N.		W.	
Jones, Henry	1	Newton, Jared	1	Wells, Wm.	1
Judson, Daniel	1			Wilson, George	1

LEGER A.

PAGE 1.

Edward P. Nixon, New York, June 1, 1833.

1833.	D. B. Folio			$	cts
		Dr. George Wilson,			
June 1.	1.	To Sundries, - - -		112	20
June 1.	1.	Dr. Henry Jones, - -			
		To Sundries, - - - -		160	70
June 2.	1.	Dr. Charles Patch, -			
		To Cash, - - -	$327,09		
		To one hogshead of gin,	124,02	451	11
June 2.	1.	Dr. John Dodson, - -			
		To Sundries, - - - -		48	75
June 3.	1.	Dr. William Wells, -			
		To Sundries, - - - -		633	00
June 6.	1.	Dr. John McNeill, -			
		To Sundries, - - - -		417	10
June 6.	1.	Dr. Daniel Judson, -			
		To Sundries, - - - -		92	00
June 8.	1.	Dr. Jared Newton, -			
		To Sundries, - - - -		58	76

LEGER A. PAGE 1.

Edward P. Nixon, New York, June 1, 1833

1833.	D. B.	Cr. - - -		$	cts
June 5.	2.	By cash $100 : his note $12,20		112	20
		Cr. - - -			
June 3.	2.	By Cash, - - - -		160	70
		Cr. - - -			
June 6.	1.	By Cash, - - -	$400,00		
		By his note, - - -	51,11	451	11
		Cr. - - -			
Jun 10.	2.	By Cash, - - -	$24,50		
		By Sundries, - -	14,25	48	75
		Cr. - - -			
June 5.	2.	By Cash in full, - - - -		633	00
		Cr. - - - -			
Jun 10.	2.	By Cash, - - -	$100		
		By bal. trans. to new act. -	317,10	417	10
		Cr. - - -			
June 8.	2.	By Sundries, - -	$27,78		
		By bal. trans. to new act. -	64,22	92	00
		Cr. - - -			
Jun 10.	2.	By Cash, - - -	$37,50		
		By error of account, -	21,26	58	76

CASH BOOK.

This book records the amount of Cash received and paid out each day.

The Cash is made Dr. to the amount of cash on hand, at the commencement of each day, and to all that is received during the day, and credited with the amounts paid out and with the balance on hand.

Dr. CASH. **Cr.**

1833.		$	cts	1833.		$	cts
June 1.	To Cash on hand,	327	27	June 6.	By rent for house,	427	18
„ 6.	„ J. Patrick,	47	15	„ 20.	„ Thomas Tappan,	12	90
„ 9.	„ P. Weeks,	125	09	July 1.	„ Merchandise bought,	512	10
July 3.	„ R. Lowndes,	82	12	„ 9.	„ Expenses to Boston,	80	13
„ 10.	„ T. Ames,	450	81	„ 25.	„ Cash on hand,		13
		1032	44			1032	44
	Cash on hand,		13				

The following is a very convenient form for book keeping, and requires but a single book. It is probably the best form for farmers and mechanics.

J. Bell, **Dr.** **J. Bell,** **Cr.**

1833.		$	cts	1833.		$	cts
June 1.	To 5 cords of wood at $1,75 per cord,	8	75	July 6.	By shoeing horse,		
„ 6.	To one day's work,	1	00	„ 10.	By mending plough,		
July 9.	To 4bu. of rye at 62 cents,	2	48	„ 20.	By ironing wagon,		
		12	23	Aug. 1.	By Cash to balance,		

APR 17 1958